A tool for control reconfiguration based on operational data

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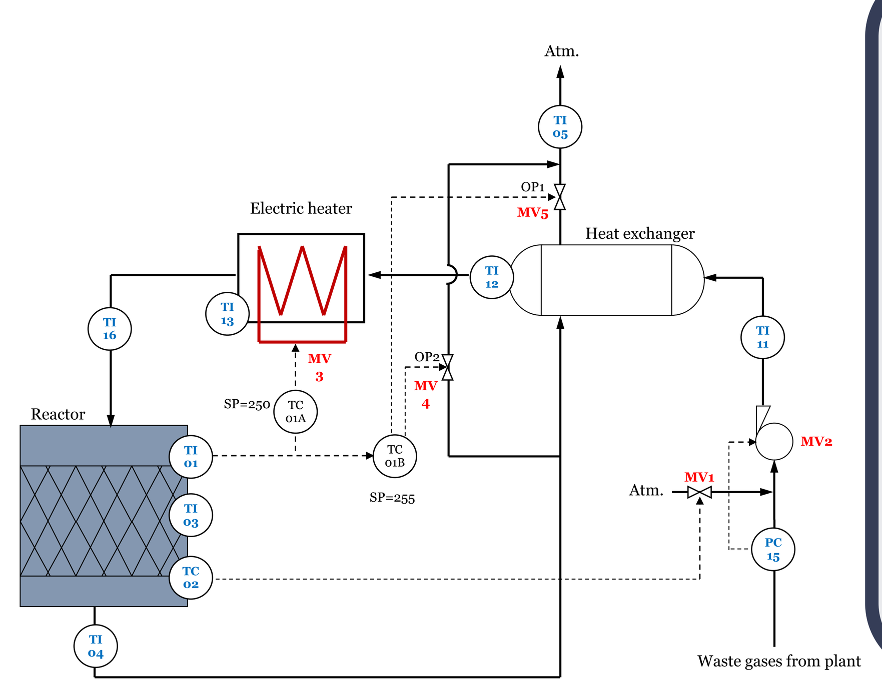
Problem description

The selection of a control configuration is often performed prior to operation based on available models and expert knowledge.



Catalytic incineration

- organic waste combustion
- regulate outlet T
- avoid use of heater
- 10 outputs
- 5 inputs
- unsatisfactory performance

A **given configuration** might not work as desired due to

- poor design, • model uncertainties, • changes in dynamics.

Yet, one can collect routine **operational data**.

Can we suggest a better control pairing from op. data?

Representations of systems and data

We consider a MIMO state-space representation, which allows for different descriptions of the data.

State-space

$$x_{k+1} = Ax_k + Bu_k + w_k \qquad y_k = Cx_k + Du_k + v_k.$$

Transfer-function

$$y_k = G(q)u_k + H(q)e_k \qquad G(q) = C\,(qI-A)^{-1}B+D \qquad e_k \triangleq \left[w_k\ v_k\right]^T$$

Impulse response

$$y_k = \sum_{i=0}^{\infty} g_k u_k + h_k e_k \qquad g_k = CA^{k-1}B$$

The input-output relations are preserved in the **predictor form**.

State-space

$$\hat{x}_{k+1} = \tilde{A}\hat{x}_k + \tilde{B}u_k + Ky_k \qquad y_k = C\hat{x}_k + Du_k + \epsilon_k.$$
$$\tilde{A} \triangleq A-KC,\ \tilde{B} \triangleq B-KD \qquad \epsilon_k \triangleq y_k - C\hat{x}_k - Du_k$$

Transfer-function

$$y_k = \tilde{G}(q)u_k + \epsilon_k \qquad \tilde{G}(q) = (I-C(qI-\tilde{A})^{-1}K)^{-1} \times (C(qI-\tilde{A})^{-1}\tilde{B}+D) \quad \boxed{=G(q)}$$

Impulse response

$$y_k = \sum_{i=0}^{\infty} \tilde{g}_k u_k + \epsilon_k \qquad \tilde{g}_k = C\tilde{A}^{k-1}\tilde{B} + \sum_{i=1}^{k} C\tilde{A}^{i-1}K\tilde{g}_{k-i}$$

The predictor form presents some advantages such as

- residuals are white, • even in closed-loop, • stable representation.

Interaction measures

A number of interaction measures can be used for control configuration.

Example: $A = \begin{bmatrix}-1 & 0\\ 0 & -2\end{bmatrix}, B = \begin{bmatrix}1/2 & 1\\ 0 & 1\end{bmatrix}, C = \begin{bmatrix}2 & 0\\ 0 & 1\end{bmatrix}, G(s) = \begin{bmatrix}\frac{1}{s+1} & \frac{2}{s+1}\\ 0 & \frac{1}{s+2}\end{bmatrix}$,

RGA

$RGA(0) = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

- gain based
- choose close to I
- single frequency
- decentralized control
- relates to CL stability
- delay dependent

Participation Matrix

$\Phi = \begin{bmatrix}0.19 & 0.76\\ 0 & 0.05\end{bmatrix}$

- Gramian based
- choose sum close to 1
- all frequencies
- structure preserving
- realization dependent
- delay dependent

Norm-2

$\Sigma_2 = \begin{bmatrix}0.27 & 0.54\\ 0 & 0.19\end{bmatrix}$

- $\mathcal{H}_2$-norm based
- choose sum close to 1
- all frequencies
- structure preserving
- realization independent
- delay independent

The Σ_2 interaction measure

It is defined as $[\Sigma_2]_{ij} = \frac{\|G_{ij}\|_2}{\sum_{i,j}\|G_{ij}\|_2}$ and is based on the $\mathcal{H}_2$-norm,

$$\begin{aligned}\|G\|_2^2 &= \frac{1}{2\pi}\int_{-\pi}^{\pi} \operatorname{tr} G^*(e^{j\nu})G(e^{j\nu})d\nu = (\text{area under Bode}) = \|g\|_2^2 = (\text{energy})\\ &= \sum_{k=0}^{\infty}\operatorname{tr} g_k^T g_k = \sum_{k=1}^{\infty}\operatorname{tr}(CA^{(k-1)}B)^T CA^{(k-1)}B + D^TD = /\text{Gramians } P,Q/\\ &= \boxed{\operatorname{tr}(B^TQB+D^TD) = \operatorname{tr}(CPC^T+DD^T)} = /\text{equivalence}/ = \|\tilde{G}\|_2^2 = \boxed{\|\tilde{g}\|_2^2}.\end{aligned}$$

Can be found using different approaches. Here we use estimates of $\tilde{g}_k$ (Markov parameters based) and of A, B, C, D (realization based).

Estimation of Σ_2 from IO data

The stability of the predictor form allows for the approximation,

$$\begin{aligned}y_{k+p} &= C\left(\tilde{A}^p x_k + \sum_{i=1}^{p}\tilde{A}^{p-i}\tilde{B}u_{k+i-1} + \sum_{i=1}^{p}Ky_{k+i-1}\right) + Du_{k+p} + \epsilon_{k+p} \approx \boxed{/\tilde{A}^p \approx 0/}\\ &\approx \sum_{i=1}^{p}C\tilde{A}^{p-i}\tilde{B}u_{k+i-1} + Du_{k+p} + \sum_{i=1}^{p}CKy_{k+i-1} + \epsilon_{k+p} = \Theta_p \underbrace{\begin{bmatrix}u_{k:k+p}^T\\ y_{k:k+p-1}^T\end{bmatrix}}_{\triangleq z_k} + \epsilon_{k+p}\end{aligned}$$

This is a **high-order VARX**, model can be found with **least squares**.

From the Markov parameters

Solve: $\widehat{\Theta}_p = \arg\min_{\Theta_p} \|Y_{p:N} - \Theta_p Z_{p:N}\|_2^2$

- Σ_2 found from $\widehat{\Theta}_p$
- known statistical behavior
- recursive solutions

Given the states and measurements it is possible to form,

$$Y_{p:N} = \left[C\ D\right]\begin{bmatrix}X_{p:N}\\ U_{p:N}\end{bmatrix} + E_{p:N}, \quad X_{p+1:N-1} = \left[A\ B\ K\right]\begin{bmatrix}X_{p:N-1}\\ U_{p:N-1}\\ E_{p:N-1}\end{bmatrix},$$

which defines two LS problems. From the definition of states,

$$\begin{bmatrix}C\\ C\tilde{A}\\ \vdots\\ C\tilde{A}^{f-1}\end{bmatrix} X_{p:N} = \underbrace{\begin{bmatrix}C\tilde{A}^{p-1}\bar{B} & C\tilde{A}^{p-2}\bar{B} & \cdots & C\bar{B}\\ 0 & C\tilde{A}^{p-1}\bar{B} & \cdots & C\tilde{A}\bar{B}\\ \vdots & \ddots & \ddots & \vdots\\ 0 & & & C\tilde{A}^{f-1}\bar{B}\end{bmatrix}}_{\boxed{\text{given by } \widehat{\Theta}_p}} Z_{p:N} = \mathcal{U}_n\Sigma_n\mathcal{V}_n^T = \mathcal{U}_n\hat{X}_{p:N}.$$

From a state-space realization

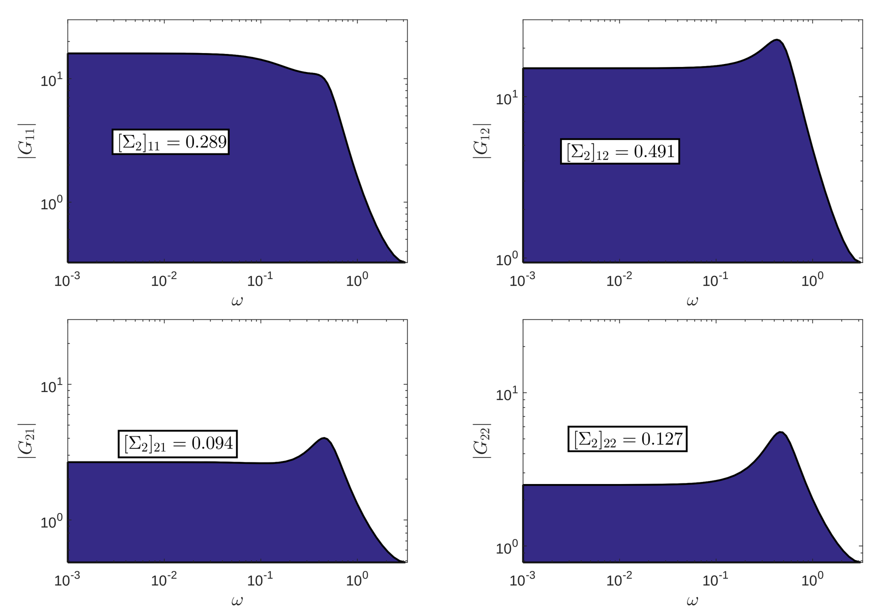
1. Find $\widehat{\Theta}_p$ from LS0
2. Find $\widehat{X}_{p:N} = \Sigma_n\mathcal{V}_n^T$ from SVD
3. Find $\widehat{C}, \widehat{D}$ and $\widehat{E}_{p:N}$ from LS1
4. Find $\widehat{A}, \widehat{B}$ and $\widehat{K}$ from LS2

- Σ_2 found from $\widehat{A}, \widehat{B}, \widehat{C}, \widehat{D}$
- SS model as a byproduct
- requires more computations
- statistical behavior difficult
- recursive solution difficult

Simulation example

$$A = \begin{bmatrix}1.5 & 1 & 0.1\\ -0.7 & 0 & 0.1\\ 0 & 0 & 0.85\end{bmatrix}, B = \begin{bmatrix}0 & 0\\ 0 & 1\\ 1 & 0\end{bmatrix},$$
$$C = \begin{bmatrix}3 & 0 & -0.6\\ 0 & 1 & 1\end{bmatrix}, F = \begin{bmatrix}0.2 & 0\\ 0 & 0.2\end{bmatrix}$$

Binary r_k, SNR$=4.15$, $N=3200$, $n=4$, $p=20$, $f=10$.



True system

$$\Sigma_2 = \begin{bmatrix}0.289 & 0.491\\ 0.094 & 0.127\end{bmatrix}$$

Markov pars.

$$\widehat{\Sigma}_2 = \begin{bmatrix}0.287 & 0.452\\ 0.142 & 0.119\end{bmatrix}$$

SS realization

$$\widehat{\Sigma}_2 = \begin{bmatrix}0.303 & 0.444\\ 0.142 & 0.111\end{bmatrix}$$

Future work

- Devise a recursive algorithm for use with real data.
- Quantify uncertainty bounds for decision making.
- Create tests for model validity based on residual analysis.