Direct Prediction-error Identification of Unstable Nonlinear Systems

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Background

Todays highly maneuverable fighter aircraft are designed to be unstable in the pitch plane in order to gain performance. Near transonic speed, i.e. close to the speed of sound, nonlinearities can occur due to aerodynamic shocks moving over the aircraft. The instability and nonlinearity have made the modern fighter aircraft dependent on control systems. In order to design the control laws, high quality simulation models are needed. A modern fighter aircraft such as JAS 39 Gripen has

a very complex flight control system. It is therefore desirable to be able to use a direct identification method on flight test data, i.e. to be able to identify the system without any knowledge of the control system.

Method

The following discrete-time state-space representation of a nonlinear output-error model is used:

$$\begin{aligned} x(t+1) &= f(x(t), u(t); \theta), \\ y(t) &= h(x(t), u(t); \theta) + e(t). \end{aligned} \tag{1}$$

A predictor of the output for the nonlinear model (1) can be written as

$$\begin{aligned} \hat{x}(t+1,\theta) &= f(\hat{x}(t,\theta), u(t); \theta) + K(t,\theta)\varepsilon(t,\theta) \\ \hat{y}(t,\theta) &= h(\hat{x}(t,\theta), u(t); \theta) \\ \varepsilon(t,\theta) &= y(t) - h(\hat{x}(t,\theta), u(t); \theta) \end{aligned} \tag{2}$$

Parameters θ are then estimated with prediction-error method (PEM). Three approaches for calculating the observer gain $K(t,\theta)$ in (2) that gives a stable predictor have been studied.

Parametrized Observer (PO) Approach

A simple approach is to let the PEM estimate the observer gain, i.e. to include K as a free time-invariant parameter in the θ vector as shown in (3).

$$\hat{x}(t+1,\theta) = f(x(t,\theta), u(t); \theta) + K(\theta)\varepsilon(t,\theta) \tag{3}$$

Extended Kalman Filter (EKF) Approach

The EKF is an extension of the Kalman Filter to nonlinear systems. The main idea is to compute $K(t,\theta)$ at each time step using a linearized model. Two versions of the EKF approach have been studied, one called EKF output-error (EKF OE) approach and second called EKF innovation form (EKF IF) approach.

EKF OE Approach

EKF OE approach is based on the linearization of the original model (1) giving

$$\begin{aligned} x_{lin}(t+1) &= A(t,\theta)x_{lin}(t) + B(t,\theta)u(t), \\ y_{lin}(t) &= C(t,\theta)x_{lin}(t) + e(t) \end{aligned} \tag{4}$$

which is then used in the EKF calculations together with (1).

EKF IF Approach

An alternative is to rewrite (4) on innovation form giving

$$\begin{aligned} \hat{x}_{lin}(t+1) &= A(t,\theta)\hat{x}_{lin}(t) + B(t,\theta)u(t) + \tilde{K}(t,\theta)\varepsilon_{lin}(t,\theta), \\ y_{lin}(t) &= C(t,\theta)\hat{x}_{lin}(t) + \varepsilon_{lin}(t,\theta). \end{aligned} \tag{5}$$

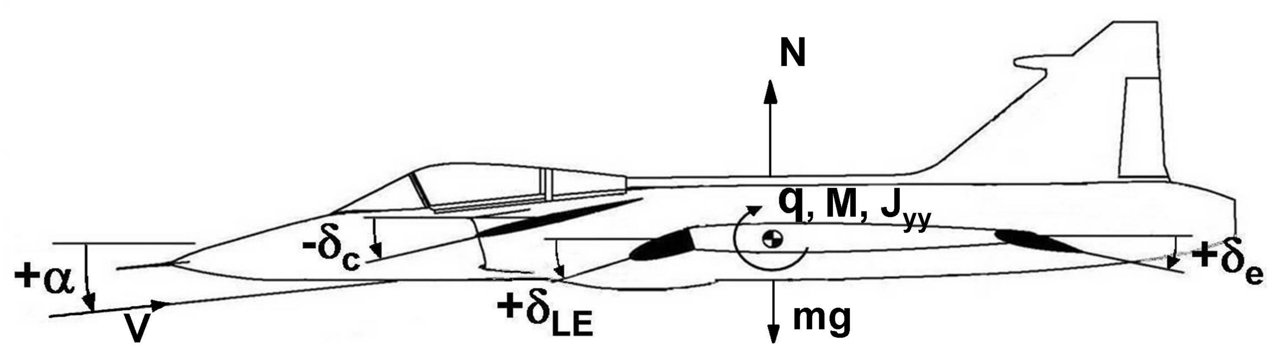
$\tilde{K}(t,\theta)$ is calculated with time-discrete algebraic Riccati equation which is then used to define system noise used in the EKF calculations.

Estimation model

To get an estimation model, the equations of motion are written as

$$\begin{aligned} \dot{\alpha}(t) &= (1/mV)\cdot N_{Aero}(\alpha(t), q(t), \delta_e(t), \delta_c(t), \delta_{LE}(t)) \\ \dot{q}(t) &= (1/J_{yy})\cdot M_{Aero}(\alpha(t), q(t), \delta_e(t), \delta_c(t), \delta_{LE}(t)) \\ y(t) &= \begin{bmatrix}\alpha(t) & q(t)\end{bmatrix}^T \end{aligned} \tag{6}$$

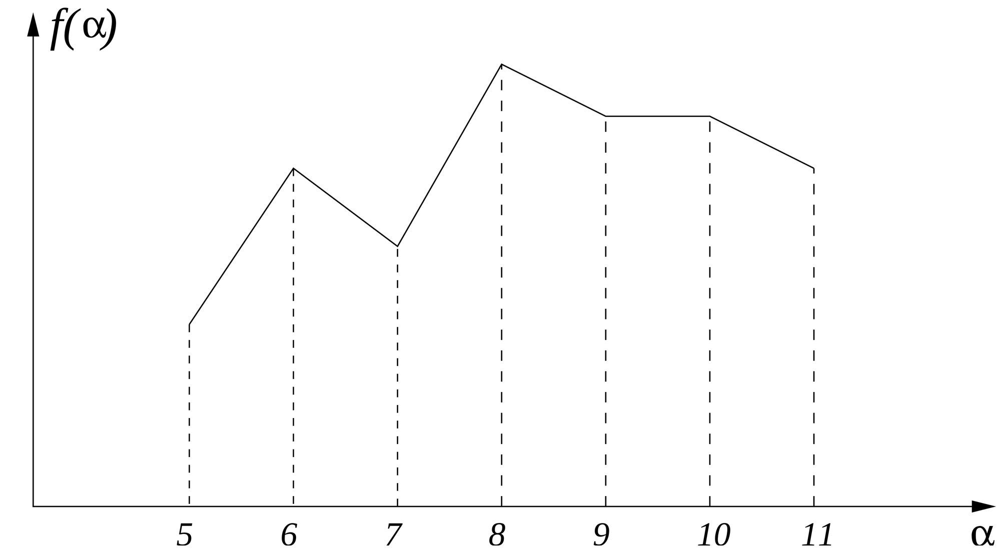
where variables are defined in the figure below.



The model is in discrete time then parametrized as

$$\begin{aligned} x(t+T) &= \begin{bmatrix} \theta_1\alpha(t) & \theta_2 q(t) \\ f(\alpha(t), \theta_{10}, \dots, \theta_{21}) & \theta_3 q(t) \end{bmatrix} + \begin{bmatrix} \theta_4 & \theta_5 & \theta_6 \\ \theta_7 & \theta_8 & \theta_9 \end{bmatrix} \begin{bmatrix} \delta_e(t) \\ \delta_c(t) \\ \delta_{LE}(t) \end{bmatrix}, \\ y(t) &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} x(t) + e(t). \end{aligned} \tag{7}$$

where $f(\alpha,\theta)$ is the nonlinear aerodynamic pitch stability model function which is built up as a piecewise affine function according to figure below for e.g. $\alpha = 5, \dots, 11$



and $\theta = [\theta_1, \dots, \theta_{21}]^T$ is estimated with real flight test data.

Results

Three approaches for direct prediction-error identification of unstable nonlinear systems have been studied, a directly parametrized observer approach and two approaches based on EKF. In general, all approaches show promising results since a good resemblance to the present aerodynamic model was found, but there are some biases in the results. These does not have to be entirely due to the estimators, but could to some extent come from the inaccuracies in the present aerodynamic model for the JAS 39 Gripen.