Convex Optimization approach for Time-Optimal Path Tracking of Robots with Speed Dependent Constraints

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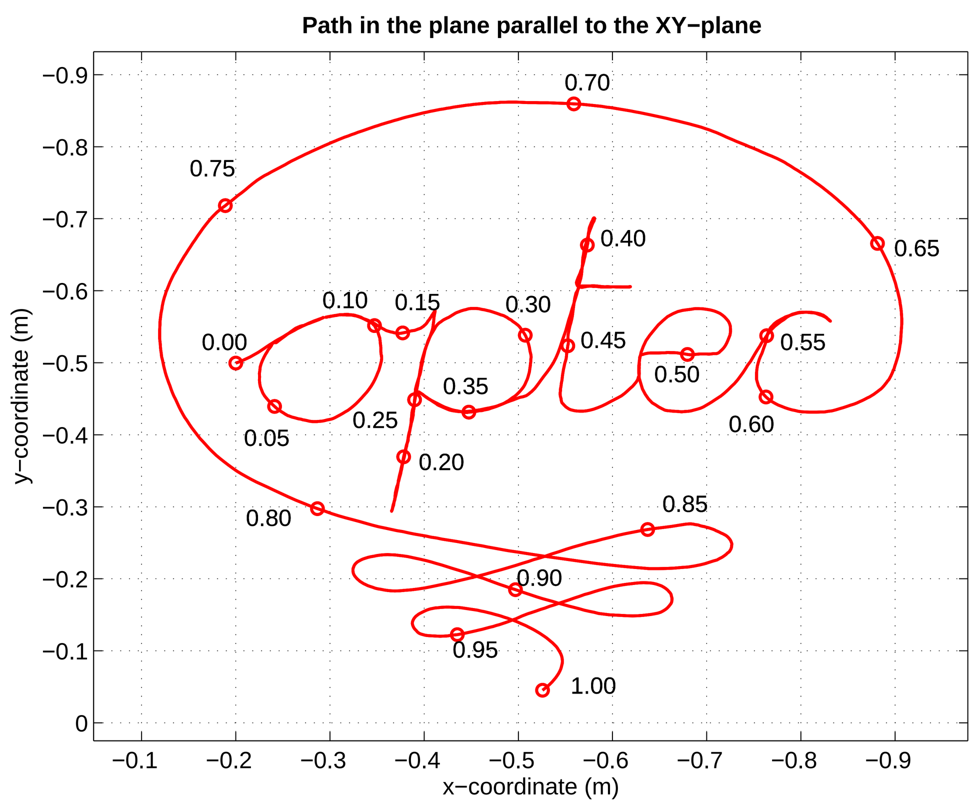
Summary

- The minimum time trajectory is computed in a highly efficient way using convex optimization.
- Compared to previous formulations of the problem, speed dependent constraints are added.
- The minimum time optimization can easily be changed to optimization with respect to energy.

Problem formulation

The path is given as a function, $q(s)$, in the joint space, where s is an index function that parameterize the path as shown below.



The trajectory $q(t)$ and its derivatives can now be expressed as,

$$\dot{q}(s(t)) = q'(s(t))\dot{s}(t),$$
$$\ddot{q}(s(t)) = q'(s(t))\ddot{s}(t) + q''(s(t))\dot{s}^2(t),$$

The time-optimal path tracking problem for the robotic manipulator can be expressed with respect to the scalar path coordinate s as,

$$\begin{aligned}
&\underset{T,s(.),\tau(.)}{\text{minimize}}\ T\\
&\text{subject to}\ \tau(t) = m(s(t))\ddot{s}(t) + c(s)\dot{s}^2(t) + g(s(t))\\
&\qquad s(0) = 0,\ s(T) = 1\\
&\qquad \dot{s}(0) = \dot{s}_0,\ \dot{s}(T) = \dot{s}_T\\
&\qquad \dot{s}(t) \geq 0,\ \dot{s}(t) \leq \bar{\dot{s}}(s)\\
&\qquad \underline{\tau}(s) \leq \tau(s) \leq \overline{\tau}(s)
\end{aligned}$$

for $t \in [0, T]$.

- The optimization problem can be reformulated into a convex optimization problem when dynamic friction is neglected.

$$\begin{aligned}
&\underset{a(.),b(.),\tau(.)}{\text{minimize}} \int_0^1 \frac{1}{\sqrt{b(s)}} ds\\
&\text{s.t.}\ \tau(s) = m(s)a(s) + c(s)b(s) + g(s)\\
&\qquad b(0) = \dot{s}_0^{\,2}\\
&\qquad b(1) = \dot{s}_T^{\,2}\\
&\qquad b'(s) = 2a(s)\\
&\qquad b(s) \geq 0\\
&\qquad b(s) \leq \bar{b}(s)\\
&\qquad \underline{\tau}(s) \leq \tau(s) \leq \overline{\tau}(s)\\
&\qquad \underline{f}(s) \leq f(s)a(s) + h(s)b(s) \leq \overline{f}(s)
\end{aligned}$$

for $s \in [0, 1]$, where $a(s)$ and $b(s)$ represents

$$b(s) = \dot{s}^2,$$
$$a(s) = \ddot{s}.$$

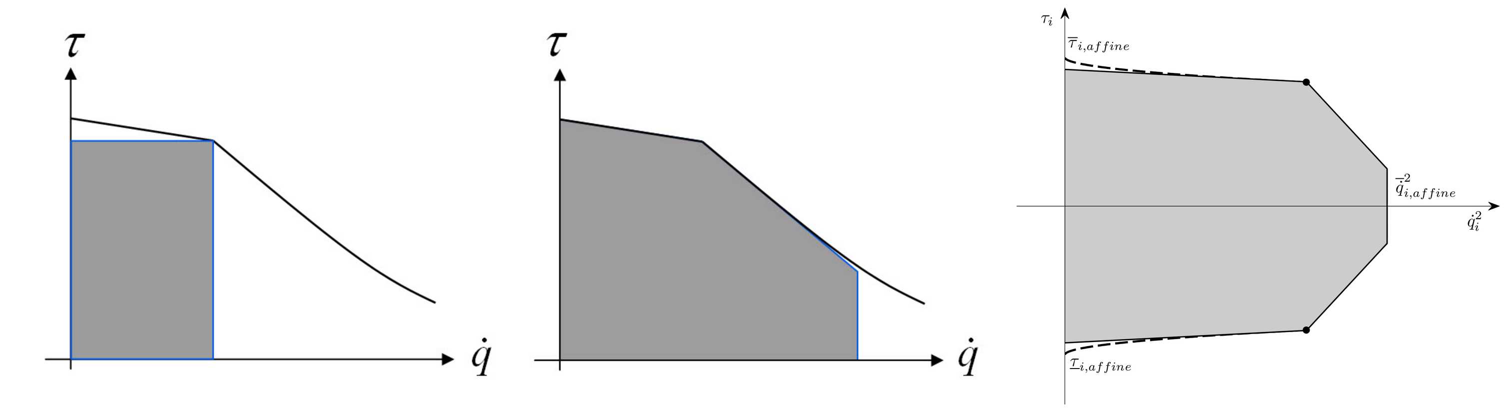
- The discretized problem can be posed as a Second Order Cone Program.

Convex speed dependent constraints

In general, two limitations have to be considered in the drive system:

1. The armature current is limited due to the heat produced in the motor,
2. The DC-voltage that can be used to drive the motors is bounded.

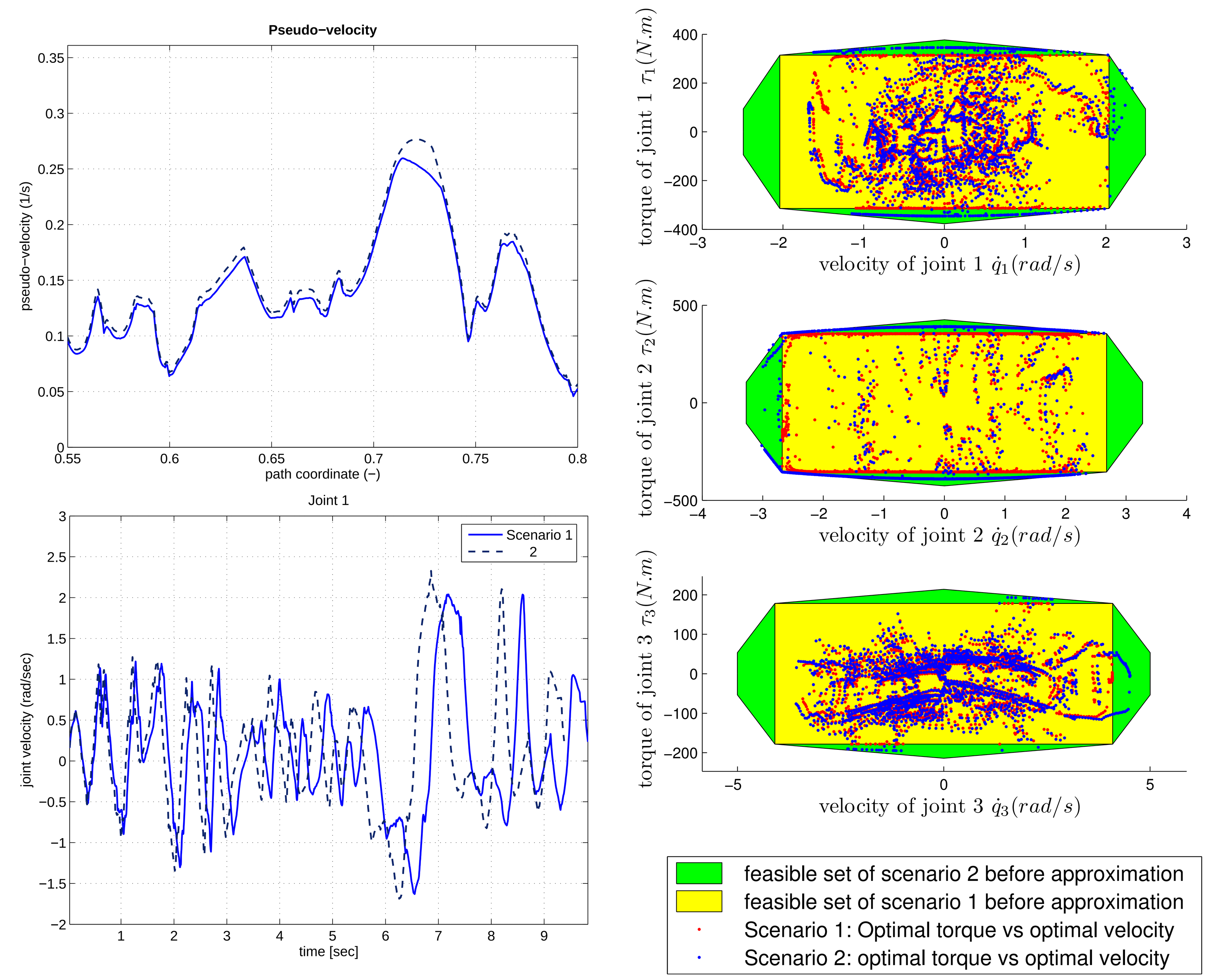
A typical torque versus speed capability specification for a brushless DC-motor is shown in the two left diagrams below.



To be able to guarantee a convex constraint, the true feasible set with respect to $\dot{q}_i$ and τ_i is approximated by a set which is convex with respect to $\dot{q}_i^2$ and τ_i. In the right most figure above the approximated constraint is illustrated for the constraint in the middle diagram.

Example

- **Scenario 1:** Rotational speed constraints are imposed on joints $1-3$. The calculated optimal joint torque versus its angular speed for this scenario is shown in yellow in the figure below.
- **Scenario 2:** An affine set of constraints is imposed on each one of the first three joints. This set of constraint is approximated by another affine set of constraint which is convex with respect to τ_i and $\dot{q}_i^2$. The figure below shows the resulting optimal joint torque versus joint speed square.



Results

- The algorithm utilizes the available additional torque and speed due to the speed dependent constraints for the three actuators.
- Some of the extra torque cannot be utilized due to the convex approximation which leads to cut-off of the non-convex part of the feasible set.
- The cycle time for the given trajectory is decreased from $9.83s$ to $9.38s$, which means a cycle time reduction of 4.6%.