Some implementation aspects of iterative learning control

Johanna Wallén, Svante Gunnarsson, Mikael Norrlöf

ELLIIT LINK-SIC LINKÖPINGS UNIVERSITET

Summary

- Implementation aspects of iterative learning control (ILC).
- Signals filtered over finite time intervals using non-causal filters.
- Theoretical analysis using matrix description of system and ILC algorithm. Simulation of a two-mass system.

Conclusion

Method of handling boundary effects of the filtering operations can have large influence over stability, convergence properties and final error level.

Purpose

- Study **implementation aspects of boundary effects** that occur in filtering operations in the ILC algorithm.
- **Matrix formulation** offers a systematic framework for analysing how this affects algorithm properties.

Filter form and matrix description

System

Linear system operating in discrete time, described by

$$y_k(t) = T_r(q)r(t) + T_u(q)u_k(t) \tag{1}$$

with reference $r(t)$, ILC input $u_k(t)$, output $y_k(t)$ at iteration k. All signals defined on a finite time interval $t \in \{0, \cdots, N-1\}$.

Parallel to description (1) a matrix description will be used, where

$$\boldsymbol{r} = \big(r(0), \dots, r(N-1)\big)^T$$

Matrix $\boldsymbol{T}_r$ formed by the pulse response coefficients $g_{T_r}(t)$ of the transfer operator $T_r(q)$, giving the Toeplitz matrix

$$\boldsymbol{T}_r = \begin{pmatrix} g_{T_r}(0) & 0 & \dots & 0 \\ g_{T_r}(1) & g_{T_r}(0) & & 0 \\ \vdots & \vdots & \ddots & \vdots \\ g_{T_r}(N-1) & g_{T_r}(N-2) & \dots & g_{T_r}(0) \end{pmatrix}$$

Using the matrix representation, the system (1) is written

$$\boldsymbol{y}_k = \boldsymbol{T}_r\boldsymbol{r} + \boldsymbol{T}_u\boldsymbol{u}_k \tag{2}$$

ILC algorithm

Two main alternatives of implementation.

Filter form:

$$u_{k+1}(t) = Q(q)\big(u_k(t) + L(q)e_k(t)\big)$$
$$e_k(t) = r(t) - y_k(t)$$

with $Q(q)$ and $L(q)$ possibly non-causal filters.

Matrix form:

$$\boldsymbol{u}_{k+1} = \boldsymbol{Q}(\boldsymbol{u}_k + \boldsymbol{L}\boldsymbol{e}_k)$$
$$\boldsymbol{e}_k = \boldsymbol{r} - \boldsymbol{y}_k \tag{3}$$

with $N \times N$ matrices $\boldsymbol{Q}$ and $\boldsymbol{L}$.

The system (2) controlled by the ILC algorithm (3) is stable iff

$$\rho\big(\boldsymbol{Q}(\boldsymbol{I} - \boldsymbol{L}\boldsymbol{T}_{\boldsymbol{u}})\big) < 1 \tag{4}$$

where $\rho(\cdot)$ denotes the spectral radius. If

$$\bar{\sigma}\big(\boldsymbol{Q}(\boldsymbol{I} - \boldsymbol{L}\boldsymbol{T}_{\boldsymbol{u}})\big) < 1 \tag{5}$$

that is, largest singular value smaller than one, the system is stable having monotone exponential convergence of $\boldsymbol{u}_k$ to $\boldsymbol{u}_\infty$.

Handling of boundary effects

Motivating example

Assume the ILC algorithm given by

$$u_{k+1}(t) = Q(q)\big(u_k(t) + L(q)e_k(t)\big), \quad L(q) = \gamma q^{\delta} \tag{6}$$

with integer $\delta > 0$ and scalar $\gamma > 0$. An assumption has to be made concerning the values of $e_k(t)$ outside the defined time interval:

Case A: $e_k(t) = 0, \quad t > N-1$
Case B: $e_k(t) = e_k(N-1), \quad t > N-1$

Filtering $e_k(t)$ using $L(q)$ corresponds to multiplying the error $\boldsymbol{e}_k$ by

$$\boldsymbol{L}_{\boldsymbol{A}} = \begin{pmatrix} 0 \dots 0 \; \gamma \; 0 \dots 0 \\ 0 \dots 0 \; 0 \; \gamma \dots 0 \\ \vdots \qquad\qquad \ddots \\ 0 \dots 0 \; 0 \; 0 \dots \gamma \\ 0 \dots 0 \; 0 \; 0 \dots 0 \\ \vdots \qquad \vdots \qquad \vdots \\ 0 \dots 0 \; 0 \; 0 \dots 0 \end{pmatrix} \qquad \boldsymbol{L}_{\boldsymbol{B}} = \begin{pmatrix} 0 \dots 0 \; \gamma \; 0 \dots 0 \\ 0 \dots 0 \; 0 \; \gamma \dots 0 \\ \vdots \qquad\qquad \ddots \\ 0 \dots 0 \; 0 \; 0 \dots \gamma \\ 0 \dots 0 \; 0 \; 0 \dots \gamma \\ \vdots \qquad \vdots \qquad \vdots \\ 0 \dots 0 \; 0 \; 0 \dots \gamma \end{pmatrix}$$

Alternatives of handling boundary effects correspond to different choices of $\boldsymbol{L}$. Influences the criteria for stability (4) and convergence (5).

Analysis using matrix description

Consider signal vector $\boldsymbol{x} = \big(x(0), \dots, x(N-1)\big)^T$. Introduce extended vector with design variable n

$$\boldsymbol{x}_e = \big(x(-n), \dots, x(0), \dots, x(N-1), \dots, x(N-1+n)\big)^T$$

Vector $\boldsymbol{x}_e$ generated by multiplication with the $N \times (N+2n)$ matrix $\boldsymbol{Q}_e$. Consider causal filter $\bar{Q}(q)$ and corresponding matrix $\bar{\boldsymbol{Q}}$. Zero-phase filtering corresponds to

$$\boldsymbol{x}_{ff} = \tilde{\boldsymbol{Q}}\boldsymbol{x} = \bar{\boldsymbol{Q}}^T\bar{\boldsymbol{Q}}\boldsymbol{x}$$

Last, truncate by removing n first and last samples of the vector by $\boldsymbol{Q}_t$. The entire filtering gives the matrix $\boldsymbol{Q}$ in (3) according to

$$\boldsymbol{y} = \boldsymbol{Q}_t\tilde{\boldsymbol{Q}}\boldsymbol{Q}_e\boldsymbol{x} = \boldsymbol{Q}\boldsymbol{x}$$

Three methods of handling boundary effects investigated:

Case I: Signal not extended.
Case II: Signal extended using first and last value.
Case III: Signal extended using linear extrapolation as in `filtfilt`

$$x(-m) = x(0) + \big(x(0) - x(m)\big)$$
$$x(N-1+m) = x(N-1) + \big(x(N-1) - x(N-1-m)\big)$$

Numerical illustration

The ILC algorithm (6) applied to a flexible two-mass system controlled by a PID-regulator.

Convergence criterion (4)

	I	*II*	*III*
A	0.895	1.007	1.101
B	0.891	0.931	1.080

Singular value condition (5)

	I	*II*	*III*
A	0.944	1.048	1.371
B	0.904	1.018	1.397