Improved real-time aerodynamic model parameter identification

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Background

Flight dynamics can often be described by a linear system

$$\dot{x} = A\,x + B\,u + Nw \quad (1a)$$

$$y = C\,x + D\,u + v \quad (1b)$$

where A and B include the properties of propulsion, mass, inertia and aerodynamics. During flight testing a real-time method could be used as decision support:

- to safely go ahead with or abort the testing
- to get enough information in data for post flight identification

Such a real-time method is presented in the book "Aircraft System Identification, Theory and Practice", by Dr. E Morelli. The method is based on a frequency domain approach that looks interesting. This has been studied and two improvements are suggested:

- Correction terms for leakage effects in Fourier transform
- Instrumental Variable (IV) method to handle noisy data

Theory

The theory is in its own rather simple and gives an easy implementation. There are two basic parts, which are described below.

1) Recursive Fourier Transformation

For the given theory eq. (1a) is given as a linear regresson $\tilde{z} = \tilde{X}\,\theta + \tilde{w}$ in the frequency domain. A recursion is applied to the transformed input u_k and output y_k. At time $t = k\,\Delta t$ this can be written as

$$\tilde{u}_k(\omega_i) = \tilde{u}_{k-1}(\omega_i) + u(k)\,e^{-j\omega_i k\Delta t}, i = 1, .., M \quad (2a)$$

$$\tilde{y}_k(\omega_i) = \tilde{y}_{k-1}(\omega_i) + y(k)\,e^{-j\omega_i k\Delta t}, i = 1, .., M \quad (2b)$$

Using transformed measurements from eq. (1b) in $\tilde{z}$ and $\tilde{X}$ gives the following equation when looking at M frequencies.

$$j\omega_i \tilde{y}_i^T = \left[\tilde{y}_i^T \; \tilde{u}_i^T\right]\left[A \; B\right]^T + \tilde{w}_i, i = 1, .., M \quad (3)$$

Suggeted improvement for the Fourier transform: Use the correct finite Fourier transform for the time derivative:

$$j\omega_i \tilde{y}_i^T \underline{- y^T(0) + y^T(t)e^{-j\omega t}} = \left[\tilde{y}_i^T \; \tilde{u}_i^T\right]\left[A \; B\right]^T + \tilde{w}_i, i = 1, .., M \quad (4)$$

2) Complex Least Squares Regression

The parameter identification of the linear regression is then solved with a complex least squares method where the estimation is given by

$$\hat{\theta} = (Re(\tilde{X}^\dagger\,\tilde{X}))^{-1} Re(\tilde{X}^\dagger\,\tilde{z}) \quad (5)$$

and an estimate of the noise variance is

$$\hat{\sigma}^2 = \frac{1}{M - n_p}(\tilde{z} - \tilde{X}\,\hat{\theta})^\dagger(\tilde{z} - \tilde{X}\,\hat{\theta}) \quad (6)$$

Suggested improvement to take care of atmospheric turbulence: use an IV approach.

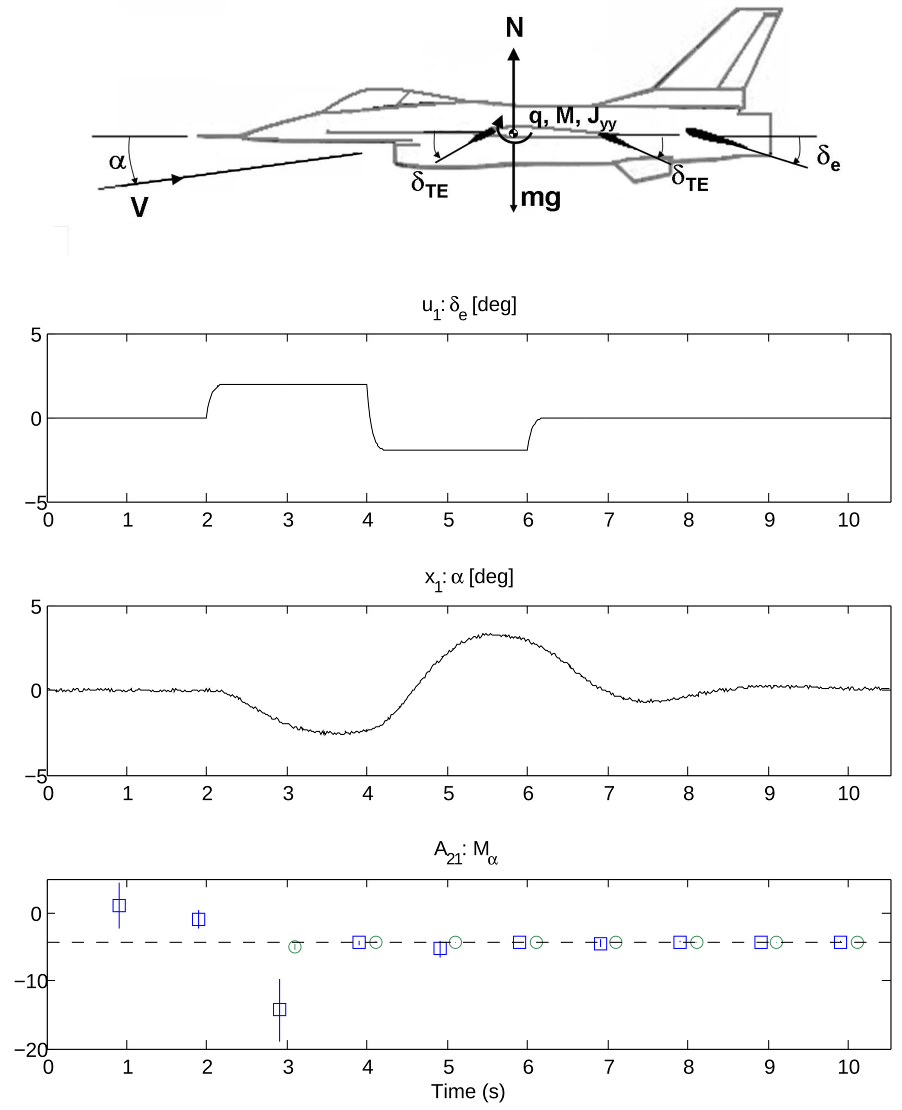
$$\hat{\theta} = (Re(\underline{\tilde{\zeta}}^\dagger\,\tilde{X}))^{-1} Re(\underline{\tilde{\zeta}}^\dagger\,\tilde{z}) \quad (7)$$

Results

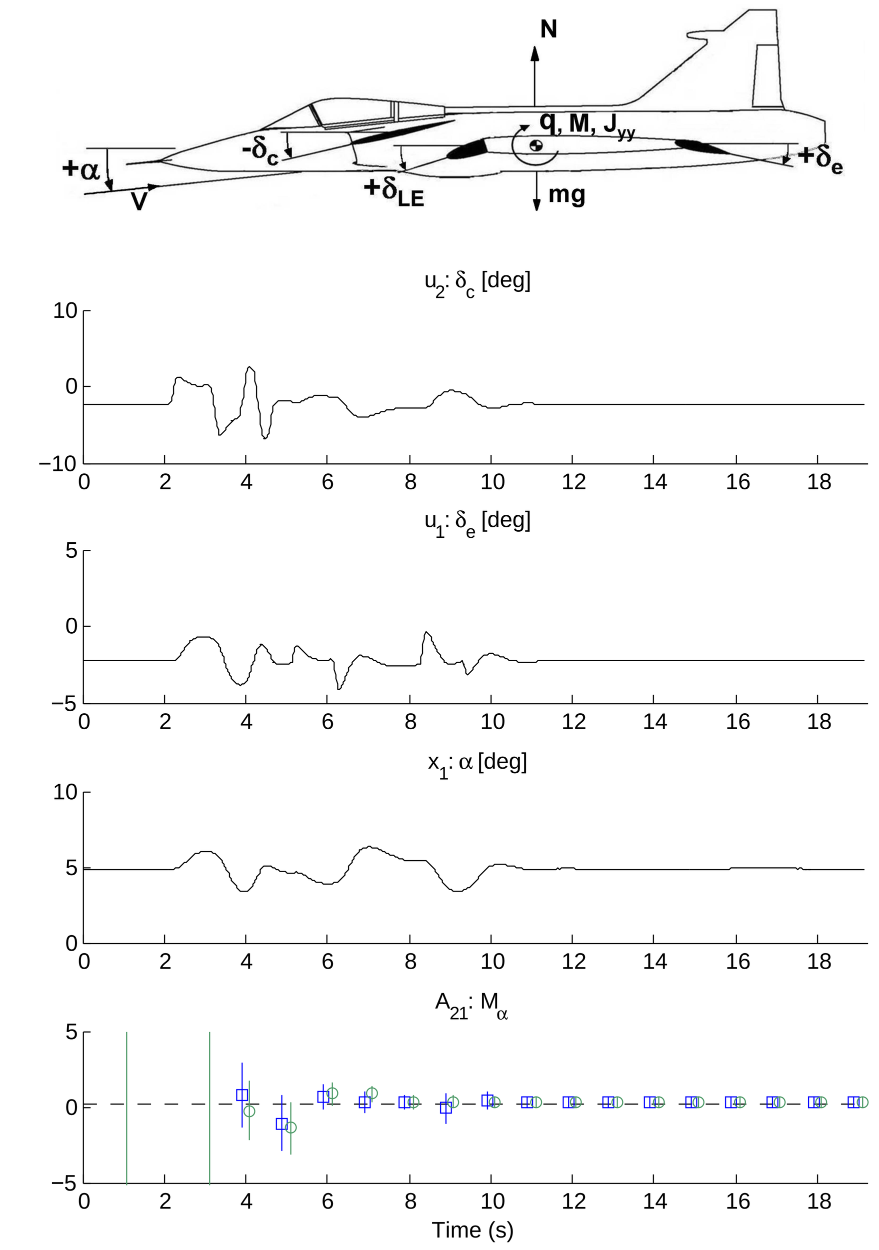
The improved method has been developed on simulated data and tested on real data. In the figures the estimation of the pitching moment is presented for the two cases.

- Using the correct finite Fourier transform improves the estimation during the excitation interval
- Using the IV approach improves the estimation when noise is present

Simulated data for an open loop system, F16:



Real data for a closed loop system, Gripen:



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