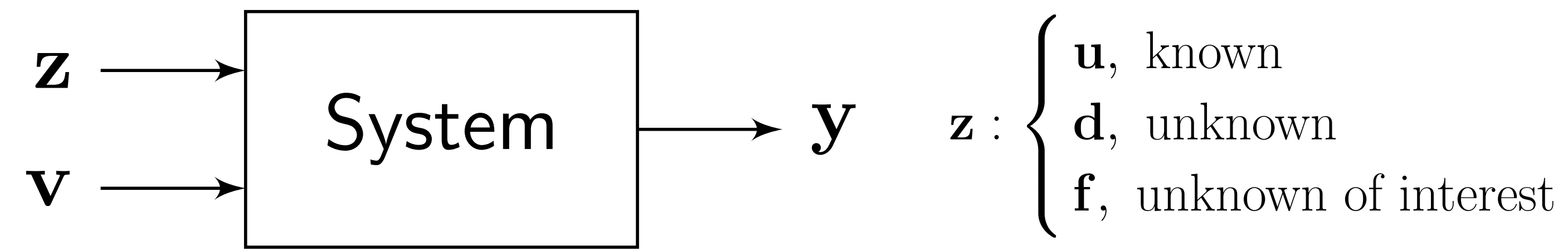


## Systems that Operate Repetitively

Consider a general system:

$\mathbf{v}$  is *random* unknown



A task  $\mathcal{U}$  is executed  $M$  times and data is collected from each execution

$$\mathbf{y}^j = [y_1^j, \dots, y_i^j, \dots, y_N^j]^T, \quad \mathbf{Y}^M = [\mathbf{y}^0, \dots, \mathbf{y}^j, \dots, \mathbf{y}^{M-1}]$$

With the purpose of monitoring  $\mathbf{y}^j$  to detect changes in  $\mathbf{f}^j$ , the following **assumptions** are made

- Faults are observable:** changes in  $\mathbf{f}^j$  affect  $\mathbf{y}^j$ .
- Regularity of  $\mathbf{y}^j$  if no fault:**  $\mathbf{y}^j$  is similar along  $j$  unless  $\mathbf{f} \neq 0$ .
- Regularity of  $\mathbf{d}^j$ :**  $\mathbf{d}^j$  is similar along  $j$ .
- Nominal data are available:** At  $j=0$ ,  $\mathbf{f}^0=0$  and  $\mathbf{y}^0$  is available.

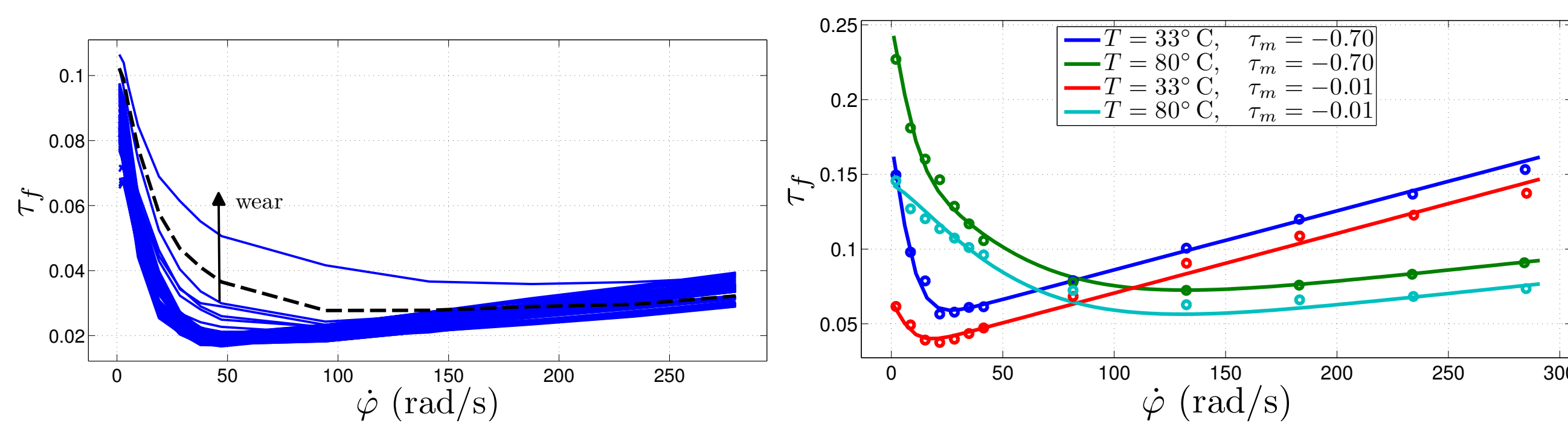
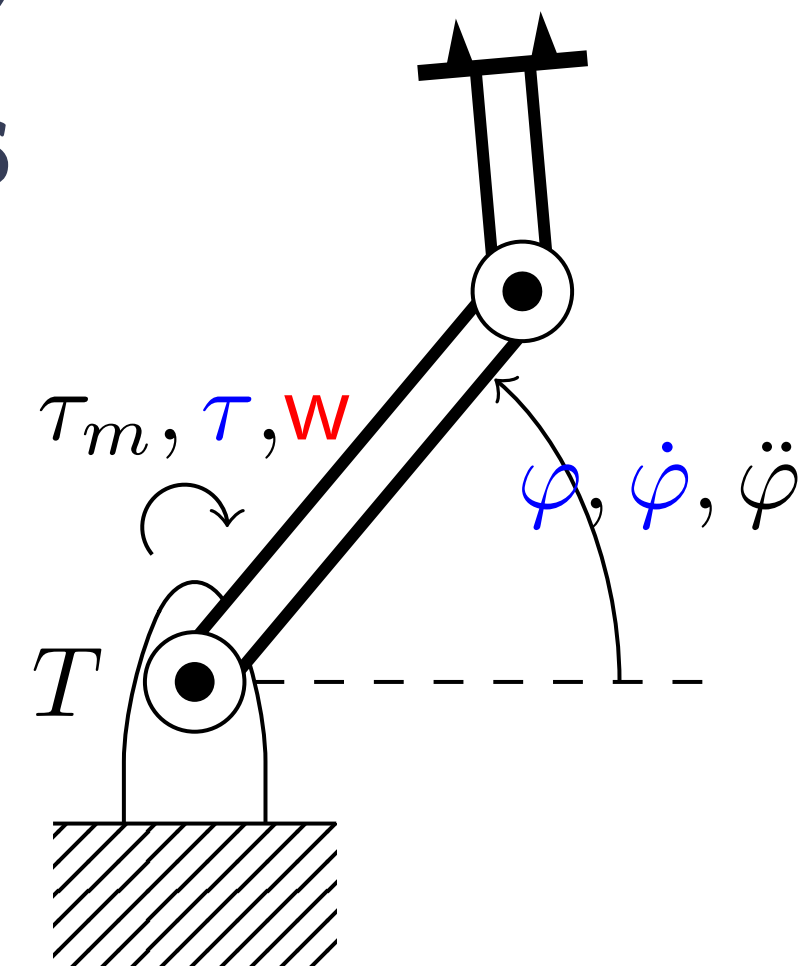
**The basic idea is then to compare  $\mathbf{y}^0$  with  $\mathbf{y}^j$ .**

## An industrial robot subject to wear and temperature changes

$$\tau = M(\varphi)\ddot{\varphi} + C(\varphi, \dot{\varphi}) + D\dot{\varphi} + \tau_g(\varphi) + \tau_s(\varphi) + \tau_f(\dot{\varphi}, \tau_m, T, \mathbf{w})$$

A task  $\mathcal{U}$  is executed regularly

$$\mathbf{y} = \tau, \quad \mathbf{f} = \mathbf{w}, \quad \mathbf{d} = \underbrace{\varphi, \dot{\varphi}, \ddot{\varphi}, \tau_m, T}_{\mathcal{U}}$$



**For more.** This work is available as a technical report at <http://www.control.isy.liu.se/~andrecb/publications/>

## Characterizing and Comparing Data

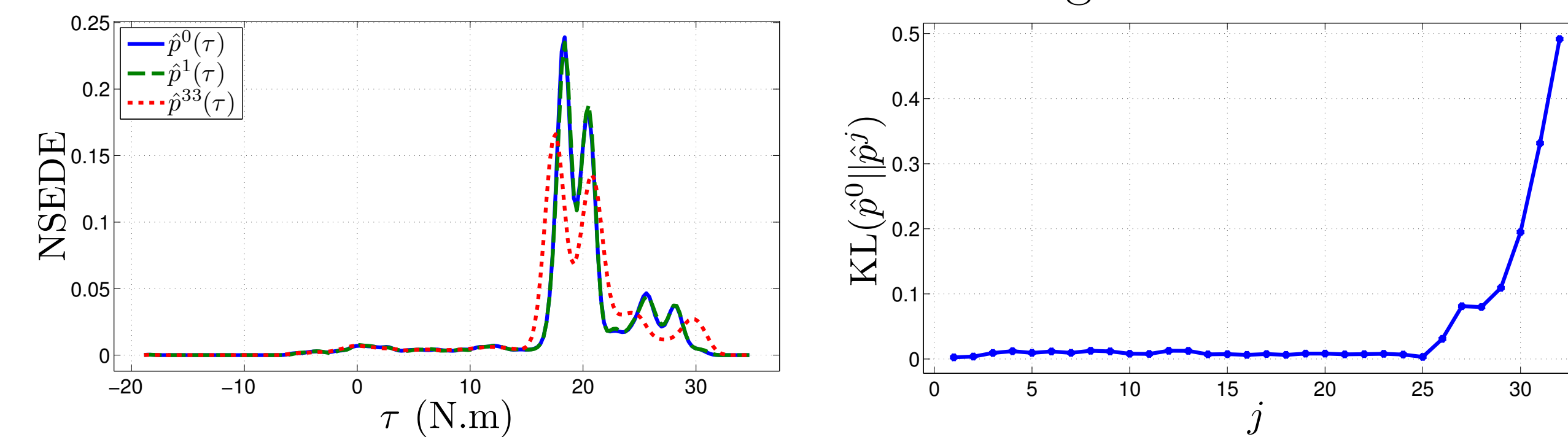
There are several possible options to characterize  $\mathbf{y}^j$ . Here, the **data distribution** is considered.

**Characterizing the data / Comparing the data**  
**Smooth Density Estimate / Kullback-Liebler distance**

$$\hat{p}^j(y) \triangleq \frac{1}{N} \sum_{i=1}^N k_h(y - y_i^j) \quad \text{KL}(\hat{p}^0 || \hat{p}^j) \triangleq D_{\text{KL}}(\hat{p}^0 || \hat{p}^j) + D_{\text{KL}}(\hat{p}^j || \hat{p}^0)$$

$$D_{\text{KL}}(\hat{p}^0 || \hat{p}^j) \triangleq - \int_{-\infty}^{\infty} \hat{p}^0(y) \log \frac{\hat{p}^j(y)}{\hat{p}^0(y)} dy$$

Real robot wear fault. Left: densities. Right: fault indicator.



- + no synchron., no ordering, little tuning, no model.
- requires  $\mathbf{y}^0$ , same  $\mathcal{U}$ , regular  $\mathbf{d}^j$ .

## Monitoring the Accumulated Changes

The objective is to relax the assumptions of known  $\mathbf{y}^0$  and regularity of  $\mathbf{y}^j$  (same  $\mathcal{U}$ ).

**No assignment of  $\mathbf{y}^0$**   
**the KL( $\cdot || \cdot$ ) is a metric**

$$\text{KL}(\hat{p}^0 || \hat{p}^j) \leq \sum_{k=1}^j \text{KL}(\hat{p}^{k-1} || \hat{p}^k)$$

Monitor increments with a CUSUM

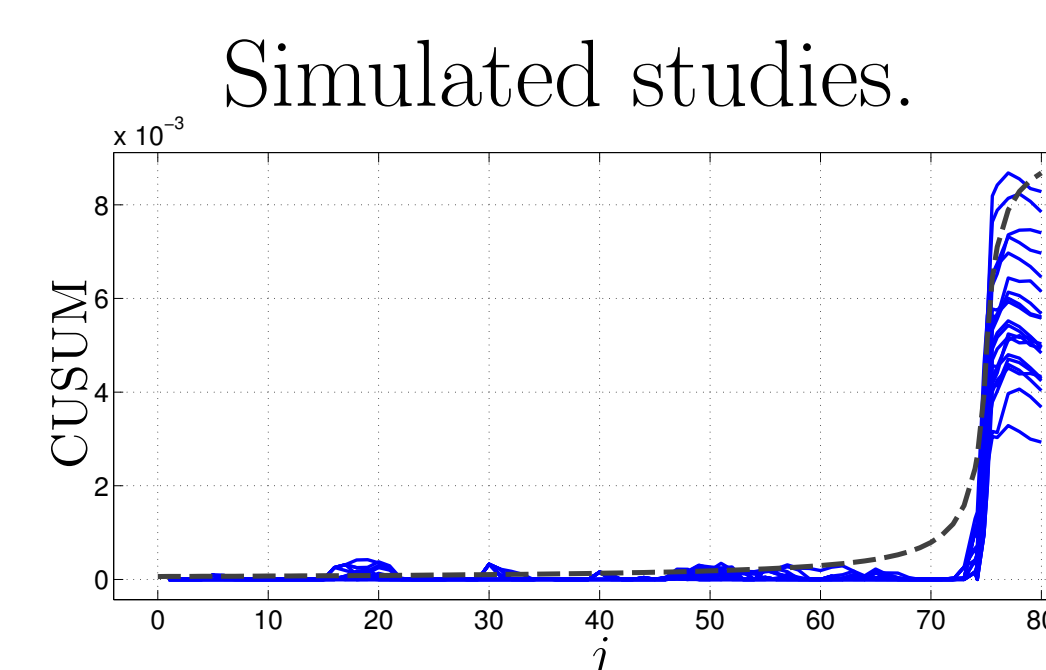
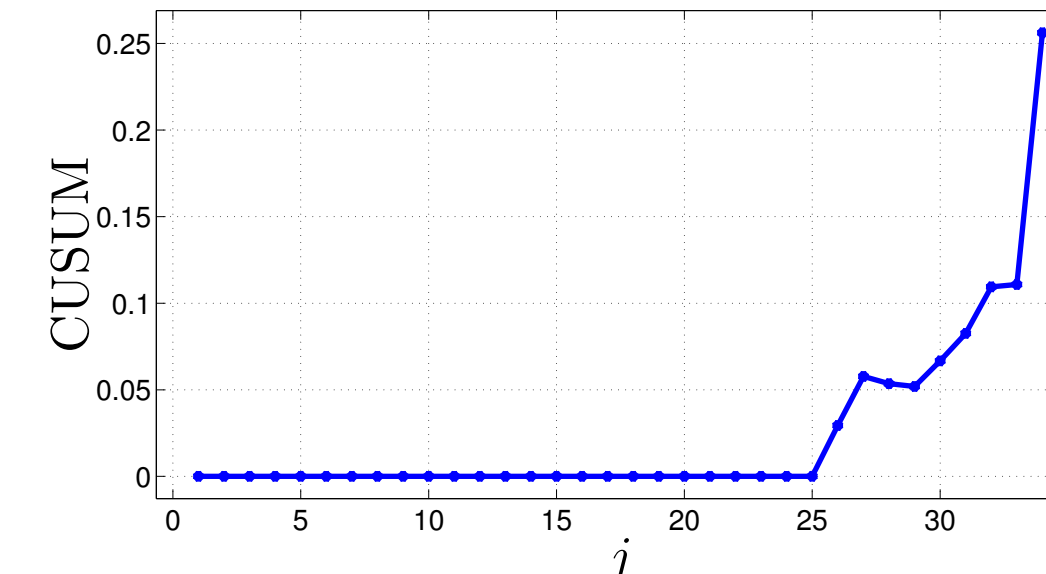
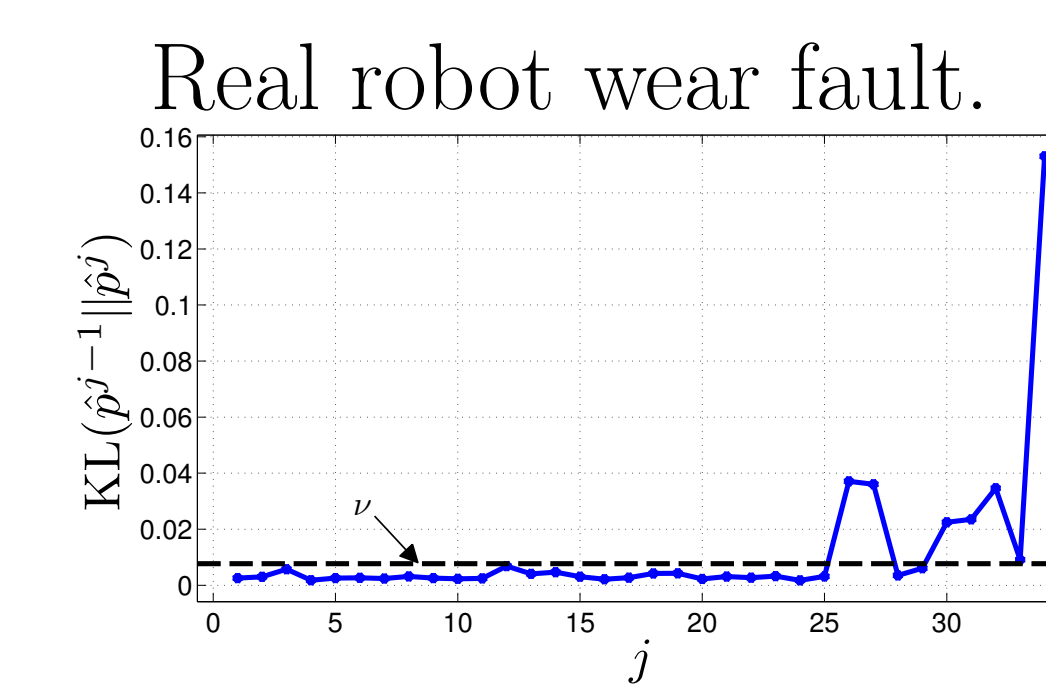
$$\begin{cases} g^j = g^{j-1} + \text{KL}(\hat{p}^{j-1} || \hat{p}^j) - \nu \\ g^j = 0 \text{ if } g^j < 0 \end{cases}$$

with  $\nu = \kappa\sigma + \mu$ .

**Handling several  $\mathcal{U}$ s**

**same! but  $\mathcal{U}^{j-1} = \mathcal{U}^j$**   
 with  $\nu^j = \kappa\sigma(\mathcal{U}^j) + \mu(\mathcal{U}^j)$

- + does not require  $\mathbf{y}^0$ , handles several  $\mathcal{U}$ s.
- regular  $\mathbf{d}^j$ , more tuning, sensitive to sampling rate.



## Reducing Sensitivity to Disturbances

In practice, it is important to cope with irregular  $\mathbf{d}$  to achieve robustness.

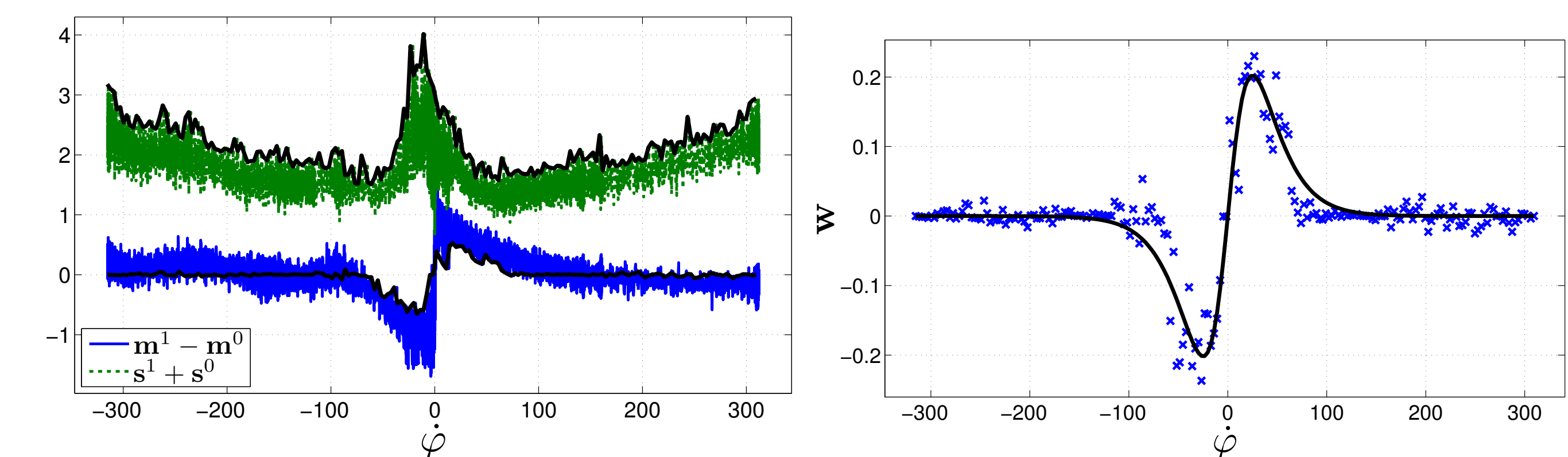
**Handling disturbances  $\mathbf{d}$**   
**apply weights to the data**

$$\bar{\mathbf{y}} \triangleq \mathbf{w} \circ \mathbf{y}$$

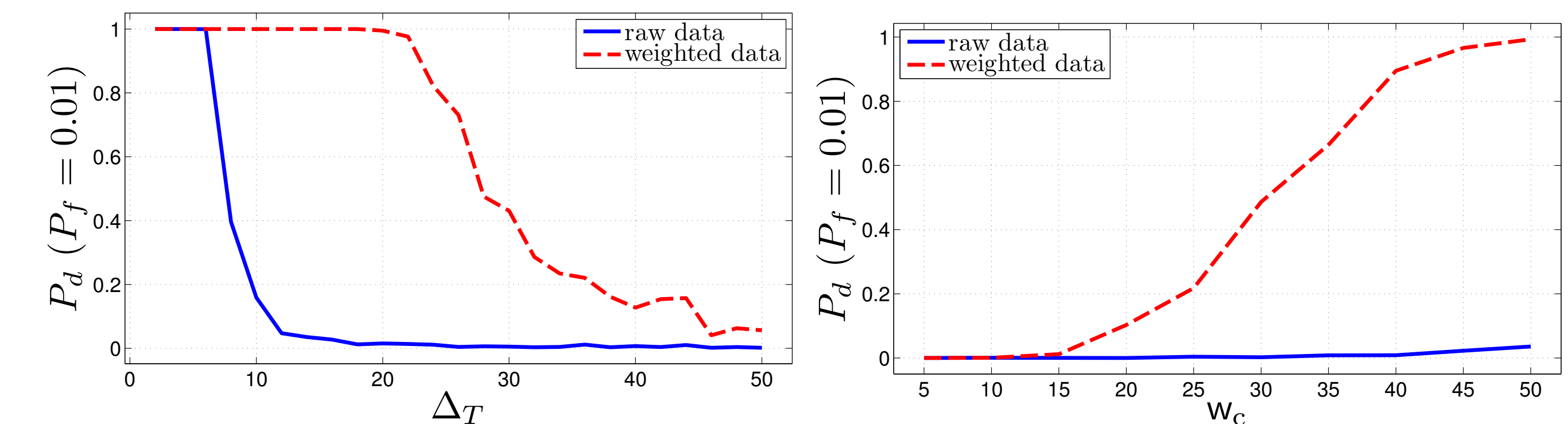
The idea is to weight data according to knowledge about disturbances and faults. Criteria similar to LDA can be used. Let  $\bar{\mathbf{m}}^k$  and  $\bar{\mathbf{s}}^k$  be the average mean and variance for the weighted faulty data,  $k=1$ , and weighted nominal data,  $k=0$ . The criterion

$$\max_{\mathbf{w}} \frac{[\bar{\mathbf{m}}^1(\mathbf{w}) - \bar{\mathbf{m}}^0(\mathbf{w})]^2}{\bar{\mathbf{s}}_1(\mathbf{w}) + \bar{\mathbf{s}}_0(\mathbf{w})} \text{ gives, } \mathbf{w} \propto (\mathbf{S}^1 + \mathbf{S}^0)^{-1}(\mathbf{m}^1 - \mathbf{m}^0)$$

For an industrial robot, optimal weights correlate with speed!



The use of weights considerably improves the detection performance.



Left: Abrupt fault of fixed size  $w_c=35$  and varying  $T$  disturbances,  $\Delta T$ .  
 Right: Abrupt fault for fixed  $\Delta T=25^\circ$  and varying fault size,  $w_c$ .

## Summary

A data-driven method for monitoring of systems that operate in a repetitive manner is proposed which

- can be used online, in a batch manner
- requires no synchronization or ordering of data sequences
- can handle disturbances
- requires nearly no knowledge of the system
- is simple and easy to implement, with little tuning