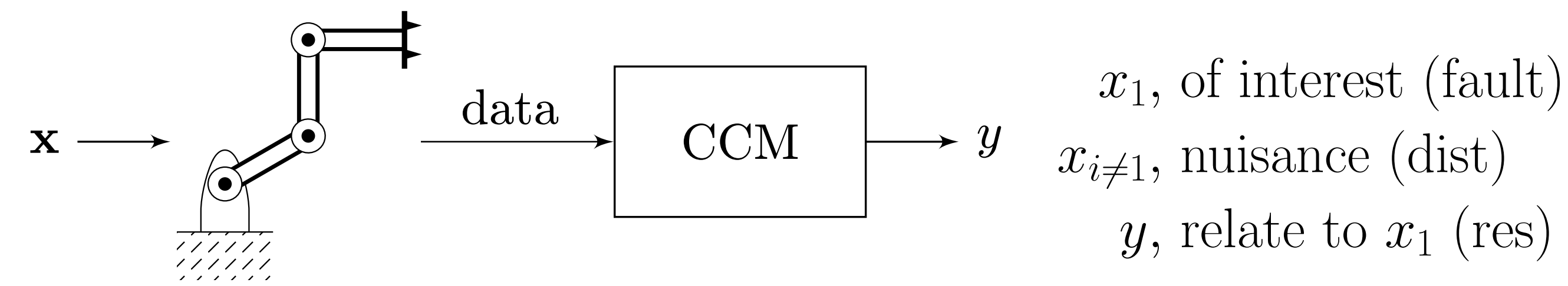


Motivation

A Condition Change Method (CCM) is an algorithm (e.g. fault detection filter) designed to process data collected from a system to generate a quantity y used to infer about a state of interest, x_1 (e.g. a fault).

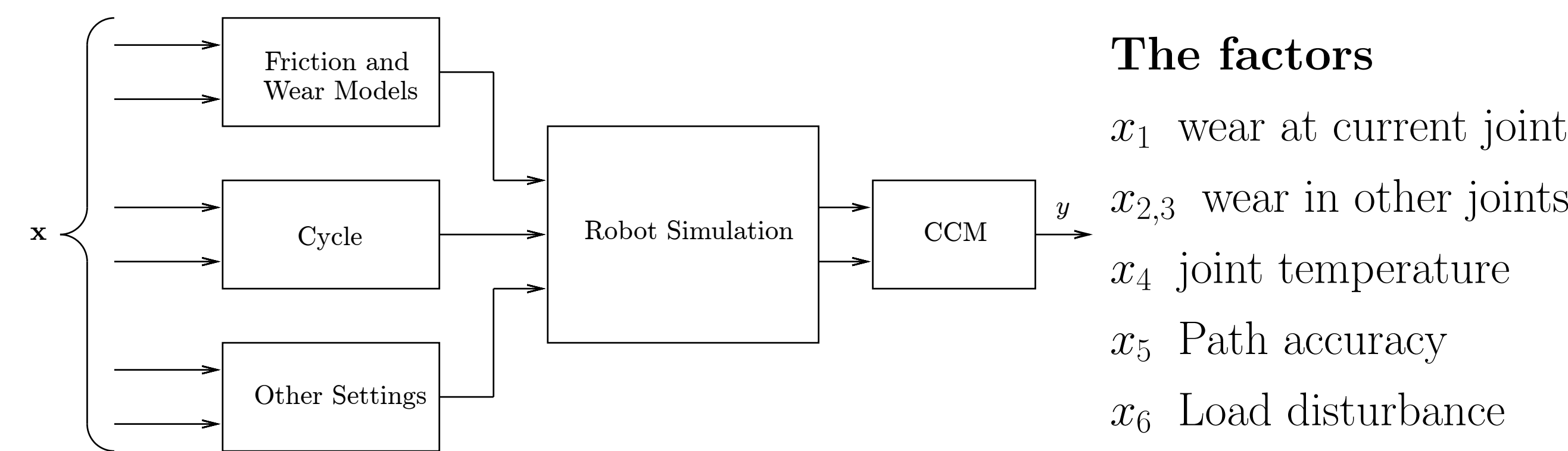


In practice, the data and therefore y is affected not only by x_1 but by $\mathbf{x} = [x_1, \dots, x_i, \dots, x_K]^T$ where $x_{i \neq 1}$ are nuisance factors (e.g. disturbances) and the ability to infer about x_1 from y might deteriorate.

Some practical issues

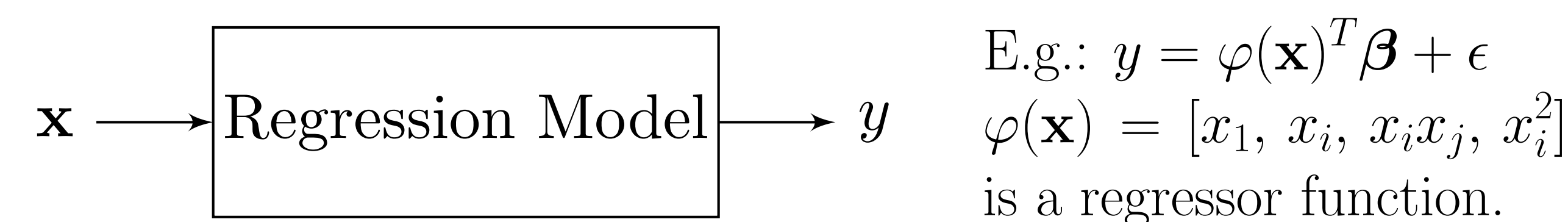
- Which factors x_i affect a CCM the most?
- How can different CCM's be compared?
- How can the performance of a CCM be quantified?
- What is the practical scope of a CCM?

Ideally, these questions should be answered using *field data* but *simulation studies* are a **cheaper and faster** solution.



Regression Models

In either case, a *regression model* can be used to **bypass the need for data**. It is a map from \mathbf{x} to y .



Several regression models might be needed depending on **design parameters**. In the robotics application, a different model is needed for each combination of CCM, robot, axis and cycle.

$$(2 \text{ robots} \times 3 \text{ axes} \times 6 \text{ cycles}) = 36 \text{ design parameters / CCM}$$

Experiment design is to choose $X = [\varphi(\mathbf{x}_1), \dots, \varphi(\mathbf{x}_j), \dots, \varphi(\mathbf{x}_N)]^T$ to find β efficiently (accurate estimate with small N).

$$\mathbf{y} = X\boldsymbol{\beta} + \boldsymbol{\epsilon}$$

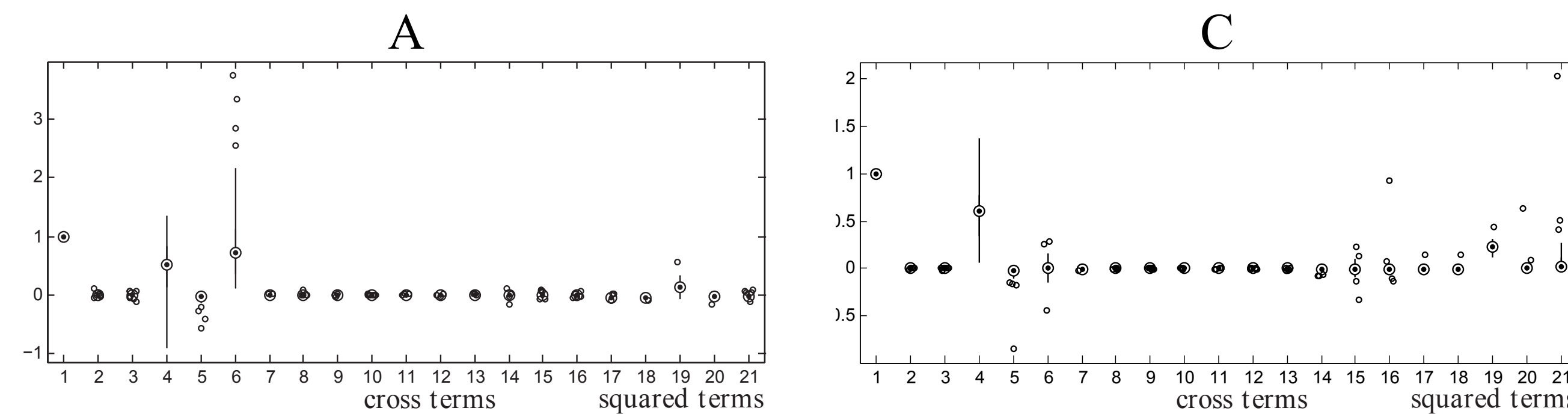
$$\hat{\boldsymbol{\beta}} = (X^T X)^{-1} X^T \mathbf{y}$$

E.g. X chosen using full factorial / composite design and $\boldsymbol{\beta}$ found through LS.

Important! x_i are **normalized**, e.g. between $[-2, 2]$. The resulting model **must** be **validated**.

Sensitivity Analysis

Investigate normalized regression coefficients β/β_1 for each CCM to conclude which factors x_i are relevant relative to the factor of interest x_1 .



In this example, CCM A shows sensitivity to both temperature and load while CCM C shows sensitivity to temperature. The boxplots are for the coefficients over all design parameters.

Signal to Noise ratio

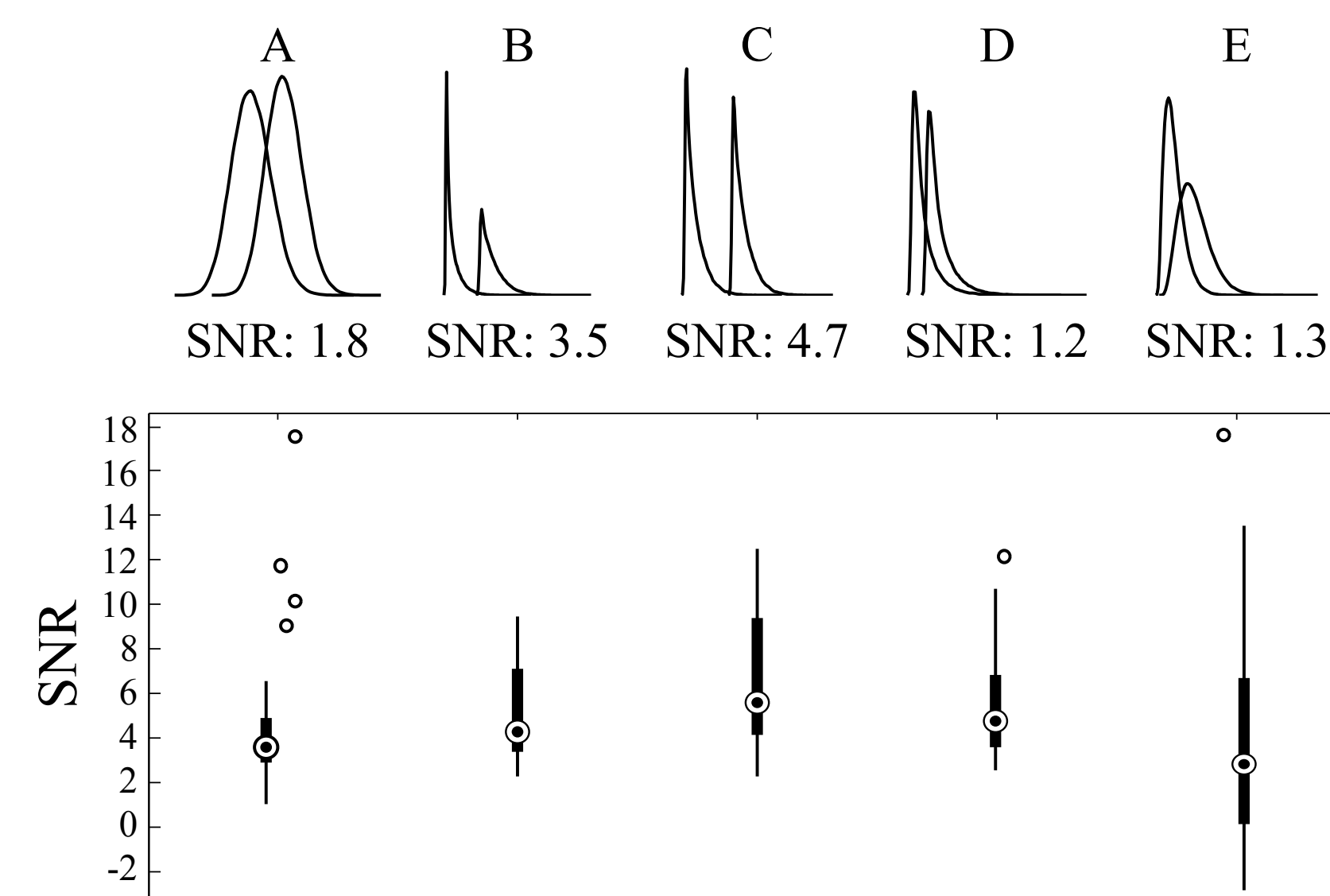
Investigate the average effect in y caused by a change δ in x_1 relative to variations in x_i , e.g. using a regression model

$$y^{(-)} = \varphi(\mathbf{x})^T \boldsymbol{\beta} - \beta_1 \delta$$

$$y^{(+)} = \varphi(\mathbf{x})^T \boldsymbol{\beta} + \beta_1 \delta$$

Using MC simulations, estimate $p(y^{(-)})$ and $p(y^{(+)})$ when $x_i \sim \mathcal{N}(\mu, \sigma^2)$.

The resulting $\text{SNR} = \mu^{(+)} / \sigma^{(+)} - \mu^{(-)} / \sigma^{(-)}$ for each CCM and cases can be used for a **performance comparison**.



MC simulations are run *efficiently* with regression models.

Boxplots are for all design parameters.

Outliers should be investigated carefully.

Determining the scope based on ROC

Build binary hypothesis tests where a change is introduced in x_1 for a random disturbance in another variable x_i

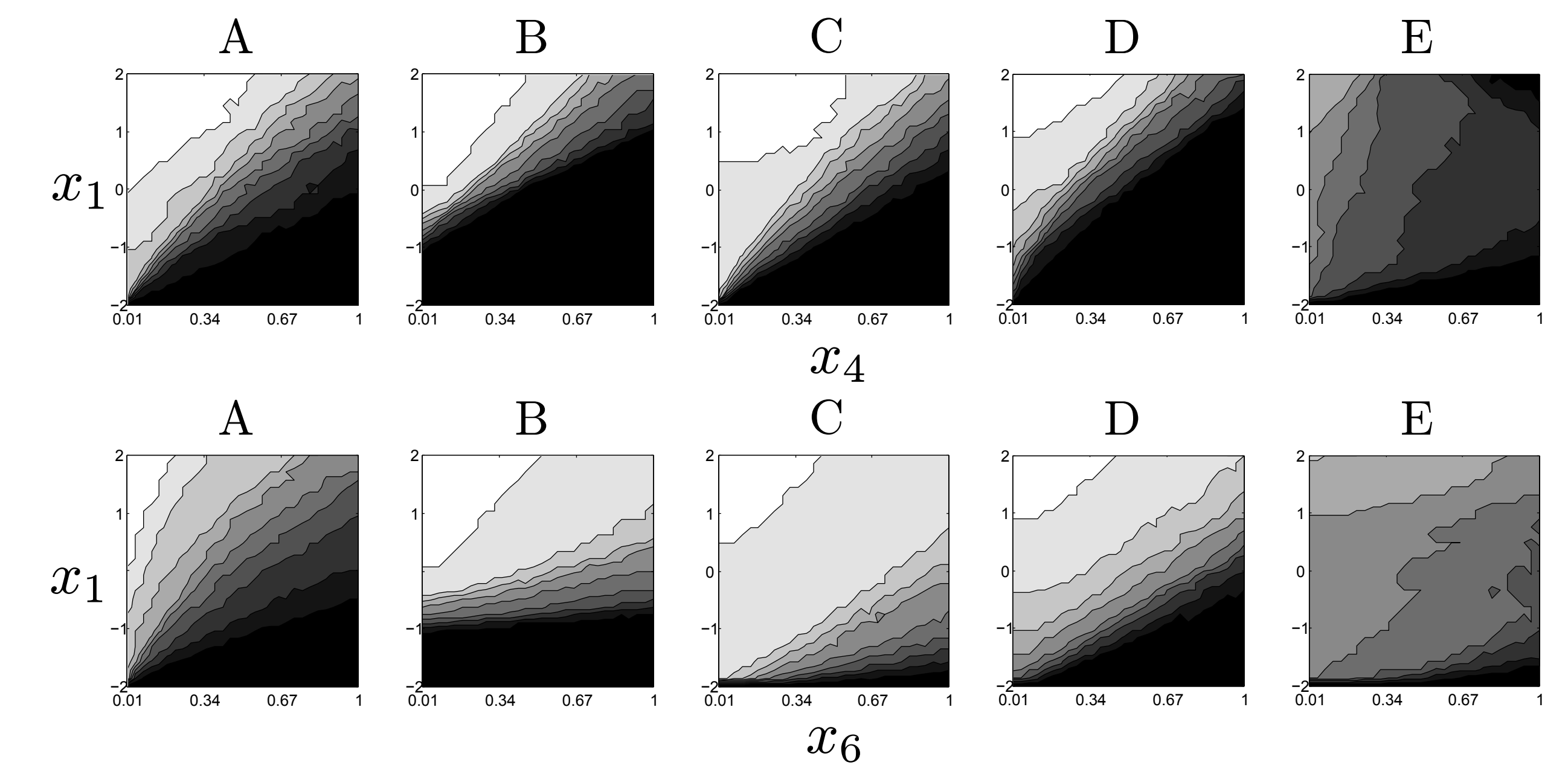
$$\mathcal{H}_0 : x_1 = -2, x_i \sim \mathcal{N}(0, \sigma^2)$$

$$\mathcal{H}_1 : x_1 = \Delta, x_i \sim \mathcal{N}(0, \sigma^2)$$

A threshold check $y \geq \bar{h}$ gives

$$P_f = \int_{\bar{h}}^{\infty} p(y|\mathcal{H}_0), P_d = \int_{\bar{h}}^{\infty} p(y|\mathcal{H}_1)$$

Evaluate P_d for a fixed $P_f = 0.01$ over different combinations of Δ and σ (in a grid), add 1 to the grid if the related $P_d \geq 0.99$. Repeat this for all design parameters and store in a **scope matrix**.



The colormap relates to how often the criterion was achieved over all design parameters (B to W, [0, 100]%). The clearer the plot, the better! In a glance, it is possible to see what combination of change in x_1 and variability in x_i restricts the usefulness of a method.

Summary

A framework for evaluation and comparison of CCMs.

- Need for data can be bypassed with *regression models*
- Important to *validate* the regression models
- *Sensitivity analyses* reveal the factors that drive the CCM
- *Outlier analyses* useful for CCM development/comparison
- The *scope matrix* shows the CCM effective regions in the factor space
- Conclusions should be taken with care (*model dependent*)