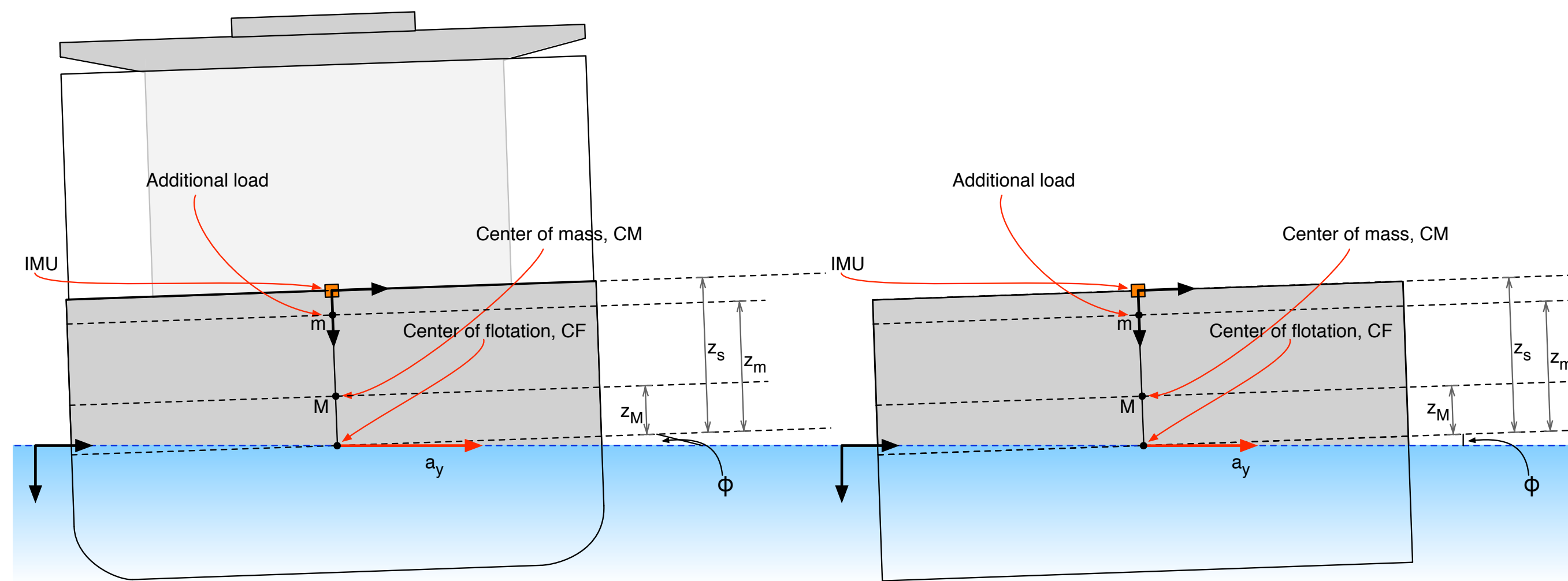


Summary

- The long term goal of this research focuses on understanding and developing methods for estimating properties of physical systems when the input of the system is unknown and only limited measurements of the system behavior are available.
- The short term goal is to estimate the change in mass and the center of mass using measurements from an inertial measurement unit.

Problem formulation

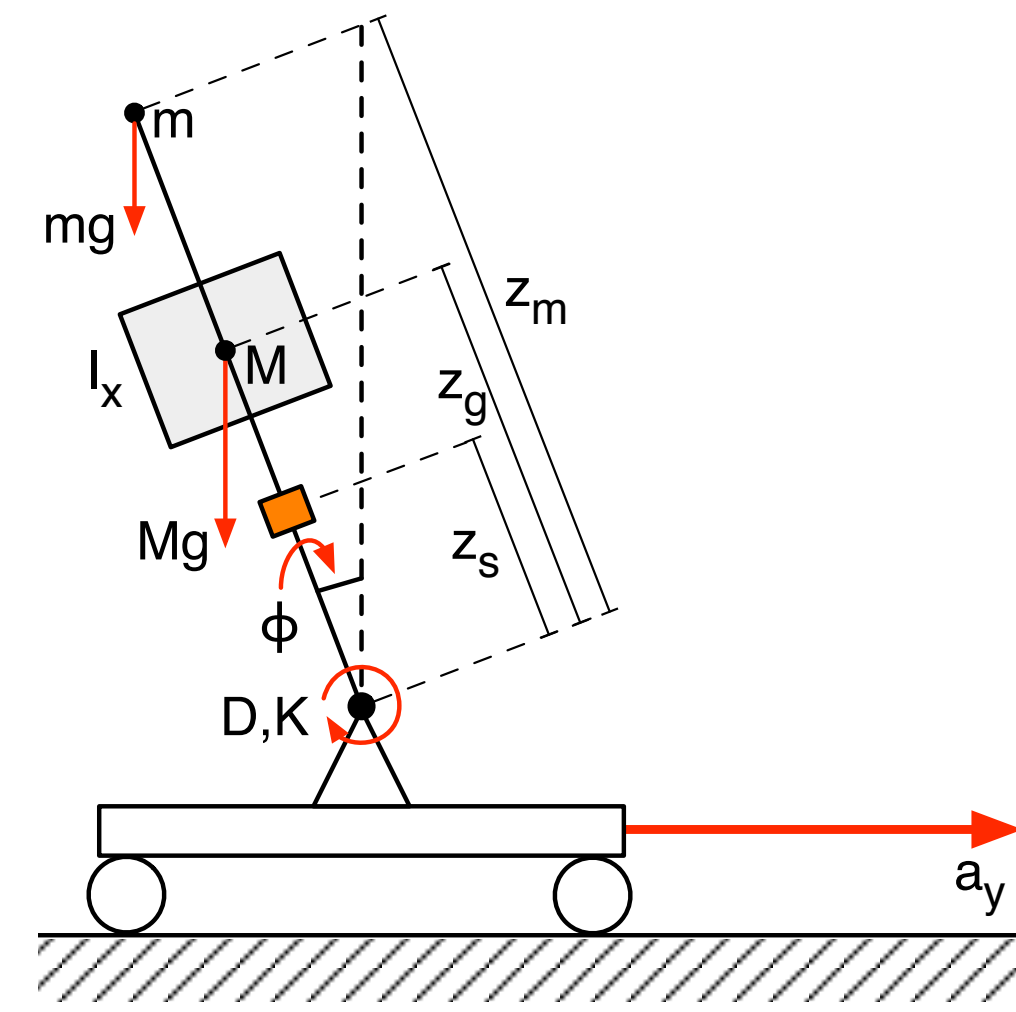
A ship is a complex structure with strong coupling between sway–roll–yaw and surge–heave–pitch. In this work we are looking at estimation of the center of mass (CM) given measurements from an inertial measurement unit (IMU). Here, the focus is on modeling parts of the system when a limited set of sensors is available.



The system is approximated as a massless pendulum with a mass M attached a distance z_g from the center of rotation (RC) and with the inertia I_x . It is hinged on a massless cart and the load is modeled as an additional mass m added to the pendulum at a distance z_m from the RC.

The external forces acting on the system are:

- An external force with the resulting unknown acceleration a_y .
- The hydrostatic restoring forces.
- Damping due to hydrodynamics effects.
- Torque disturbance v acting on the RC, e.g. waves.



The system is described by

$$\ddot{\phi} = \frac{\overbrace{-(k - Mg z_g - mg z_m)\phi}^{\text{restoring forces}} - \overbrace{d\dot{\phi}}^{\text{damping}} + \overbrace{(M z_g + m z_m)a_y + v}^{\text{disturbances}}}{I_x + M z_g^2 + m z_m^2} \quad (1a)$$

$$y = [\dot{\phi}, a_s]^T + e = [\dot{\phi}, -z_s \ddot{\phi} + g\phi + a_y]^T + e \quad (1b)$$

Proposed approach

There are two main issues with model (1)

1. The acceleration a_y affecting the RC is unknown.
 2. The disturbance from the waves v is only observed through the dynamics.
- Firstly, we propose an approach that avoids the introduction of an approximate model of a_y

$$\ddot{\phi} = \frac{-k\phi - d\dot{\phi} + (M z_g + m z_m)(a_s - e_2) + v}{I_x + M z_g(z_g - z_s) + m z_m(z_m - z_s)} \text{ and } y = \dot{\phi} + e_1 \quad (2)$$

Secondly, we suggest an **instrumental variable (IV)** approach, with the rudder angle as an instrument, to deal with the process noise and the measurement noise affecting both the input and output.

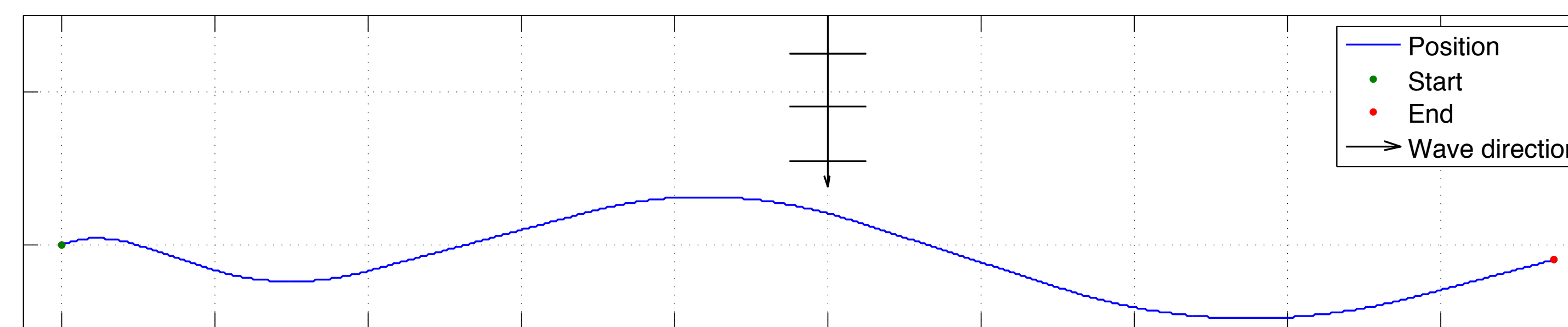
Wave-induced forces and torques

When it comes to ships, there are three major environmental forces acting on the vessel, namely currents, wind and waves. Here we focus on waves.

- The disturbance is **colored and time-varying** since the peak frequency and wave spectrum are dependent on ship's pose and speed.
- The wave-induced forces influence the ship greatly which gives a **low SNR** and forces all modes of the ship to oscillate.
- The wave-induced forces acting on the ship will be **dependent on a change in direction** and thus on the rudder angle.

Initial results with waves

A ship is traveling with the constant speed 11 m/s and the input to the system is the resulting acceleration from turning back and forth. The objective is to estimate the inertia I_x and the damping d .



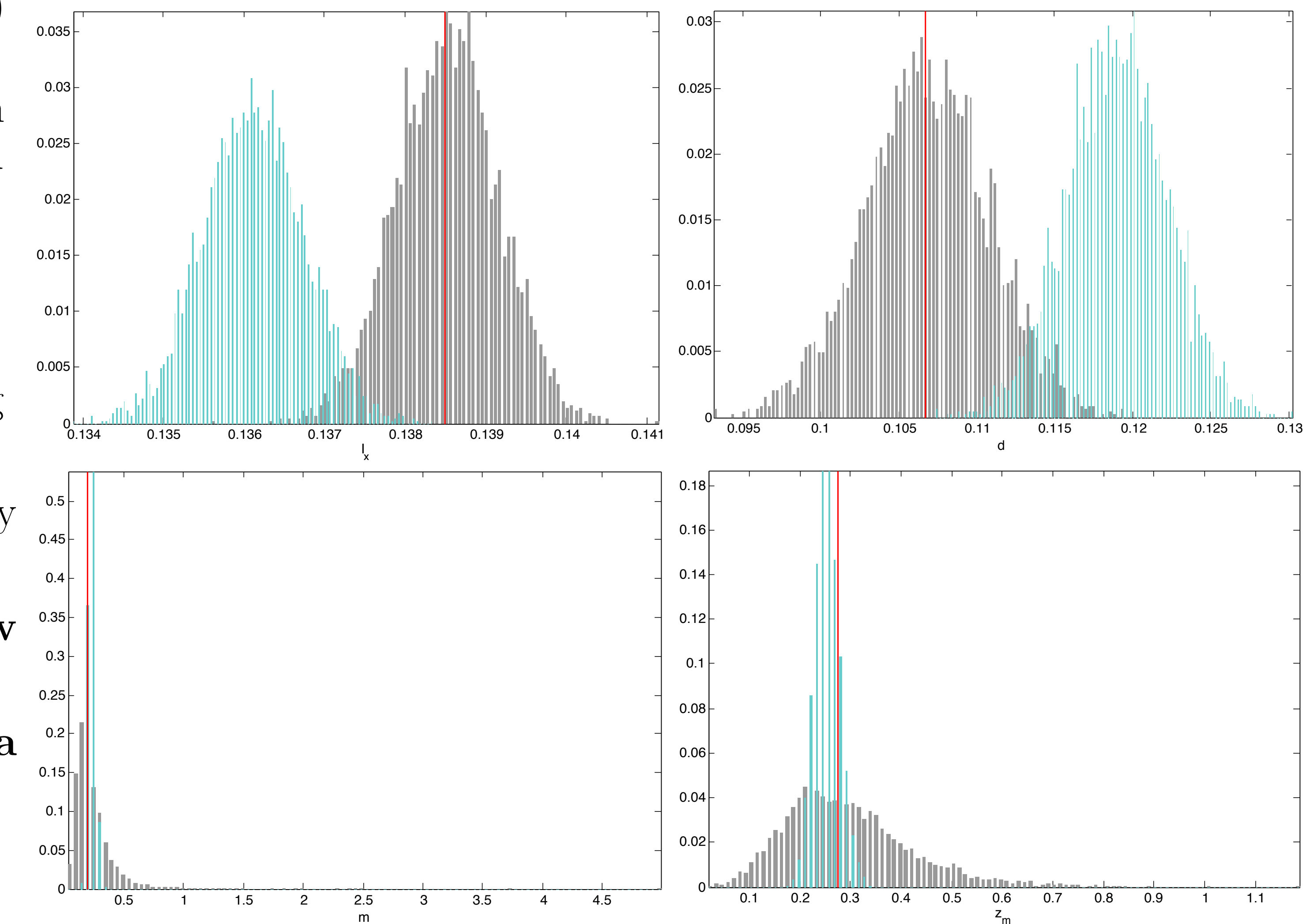
The data set is the acceleration a_s , the angular velocity $\dot{\phi}$ and rudder angle (used as an instrument). For comparison, the parameters in (2) are estimated using both the IV approach and a similar least squares (LS) approach.

Param.	True	IV Est.	IV Rel. Err.	LS Est.	LS Rel. Err.
I_x	1000	851	14.9 %	1108	10.8 %
d	5000	4343	13.1%	2409	52.9 %

The results were obtained using a dataset containing 3000 data points and the SNR was 1.16 in $\dot{\phi}$.

Bias properties

A Monte Carlo simulation with 4500 samples was carried out to study the biases caused by using the acceleration measurement as an input. As in the previous section, the parameters are also estimated using a LS method for comparison. The red line shows the true value, the grey bars show the result from the IV method and the light-blue bars show the result for the LS method.



Note that the IV method captures the mean quite well, but that the variance is increased. However, the choice of instrument is not chosen to minimize the variance.

Ongoing work

- Cooperation with Prof. Thor I. Fossen and *Centre for Autonomous Marine Operations and Systems (AMOS)* resulted in a series of tests with a model ship to collect real data in a towing tank.
- This data is supposed to be used for validation of the method and will also be used to learn which effects that cannot be neglected.

