System Identification of Flight Mechanical Characteristics

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Background

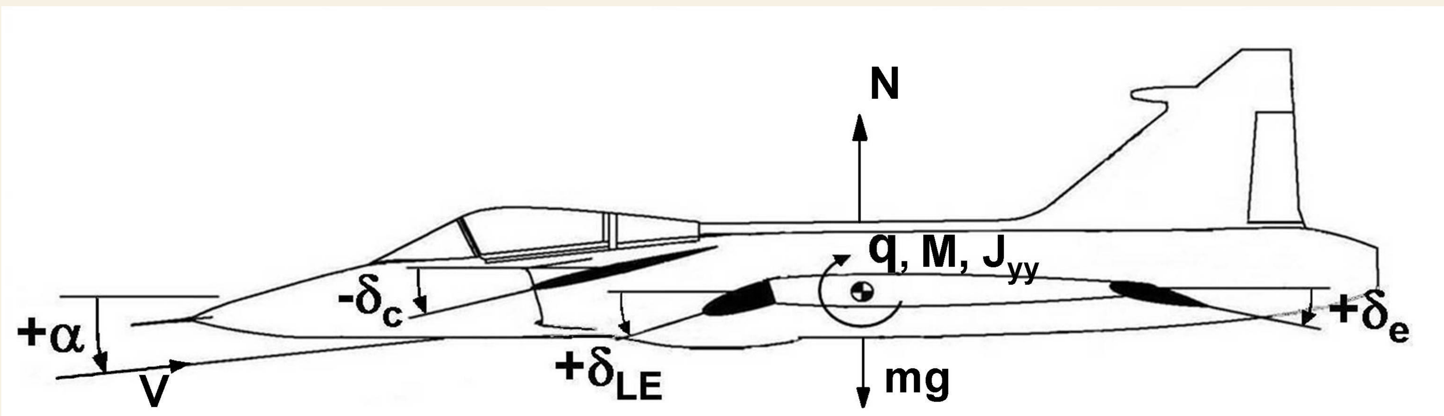
Core problem:
Identification of a nonlinear and unstable system operating in closed-loop. Interested in methods that:

- use direct identifiaction because they can easily be used on different applications.
- have no or few tuning parameters in order to get user-independent results in industry.
- can handle non-white process noise because this can appear in the applications.

Application:
Modern fighter aircraft are challenging from a system identification perspective since they work under conditions where the physical properties changes from linear to nonlinear, from unstable to stable and always operate under closed-loop conditions. The measurement noise is concidered to be white, but the process noise used in simulations is based on the Dryden atmospheric model.

Gripen

A Gripen model



A simplified model of the pitch dynamics:

$$x(k+1) = a(x(k)) + Bu(k) + w(k)$$
$$y(k) = x(k) + e(k)$$

where $x(k) = \big(\alpha(k)\ q(k)\big)^T$, $u(k) = \big(\delta_e(k)\ \delta_c(k)\big)^T$ and

$$a(x(k)) = \begin{pmatrix} Z_\alpha \alpha(k) + Z_q q(k) \\ f(\alpha(k)) + M_q q(k) \end{pmatrix}, \quad B = \begin{pmatrix} Z_{\delta_e} & Z_{\delta_c} \\ M_{\delta_e} & M_{\delta_c} \end{pmatrix}$$

Here, $f(\alpha(k))$ is a piece-wise affine function and $Z = -\mathsf{N}$.

Methods

Five approaches have been investigated:

- Three of these are based on the Prediction-Error-Method (PEM)

$$\begin{aligned} \hat{x}_{k+1}(\theta) &= f(\hat{x}_k(\theta), u_k; \theta) + K_k(\theta)\varepsilon_k(\theta), \\ \hat{y}_k(\theta) &= C\hat{x}_k(\theta), \\ \varepsilon_k(\theta) &= y_k - \hat{y}_k(\theta). \end{aligned} \tag{1}$$

$$V_N(\theta, Z^N) = \frac{1}{N}\sum_{k=1}^{N} \frac{1}{2}\varepsilon_k(\theta)^T \varepsilon_k(\theta), \tag{2}$$

where Z^N represents the N input-output measurements. An unconstrained optimization problem has to be solved to estimate θ

$$\underset{\theta}{\text{minimize}}\ V_N(\theta, Z^N). \tag{3}$$

The three approaces are different in the way they calculate the observer gain $K_k(\theta)$.

- Parameterized Observer (**PO**), ($K_k(\theta)$ part of the parameter vector)
- Extended Kalman Filter (**EKF**)
- Unscented Kalman Filter (**UKF**)

- One State Estimation method:

Here the parameter vector θ is included in the state vector x.

$$\begin{aligned} \hat{\bar{x}}_{k+1} &= \begin{bmatrix} x_{k+1} \\ \theta_{k+1} \end{bmatrix} = \begin{bmatrix} f(x_k, u_k; \theta_k) \\ w_{\theta k} \end{bmatrix} + K_k(\theta)\varepsilon_k(\theta), \\ \hat{y}_k(\theta) &= C\hat{\bar{x}}_k(\theta), \\ \varepsilon_k(\theta) &= y_k - \hat{y}_k(\theta). \end{aligned} \tag{4}$$

This is an Augmented State approach (**AUG**). The data have been filtered once using EKF to calculate the gain $K_k(\theta)$.

- One State and Parameter Estimation method:

Here the state vector x is included in the parameter vector ϑ

$$\vartheta = [x_0^T\ \dots\ x_{N-1}^T\ \theta^T]^T. \tag{5}$$

$$\begin{aligned} \hat{y}_k(\theta) &= C\hat{x}_k(\vartheta), \\ \varepsilon_k(\vartheta) &= y_k - \hat{y}_k(\vartheta). \end{aligned} \tag{6}$$

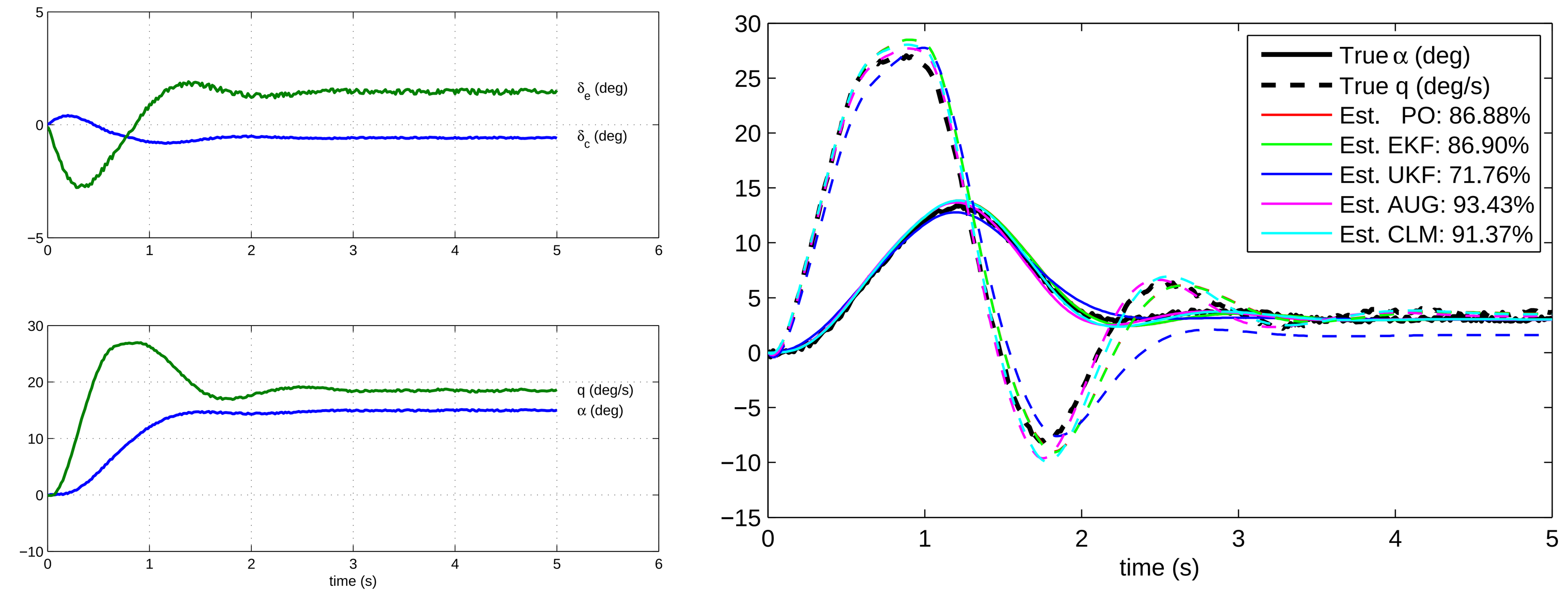
$$V_N(\vartheta, Z^N) = \sum_{k=1}^{N} \frac{1}{2}\varepsilon_k(\vartheta)^T \varepsilon_k(\vartheta), \tag{7}$$

A Constrained Levenberg-Marquardt (**CLM**) optimization problem has been solved to estimate ϑ.

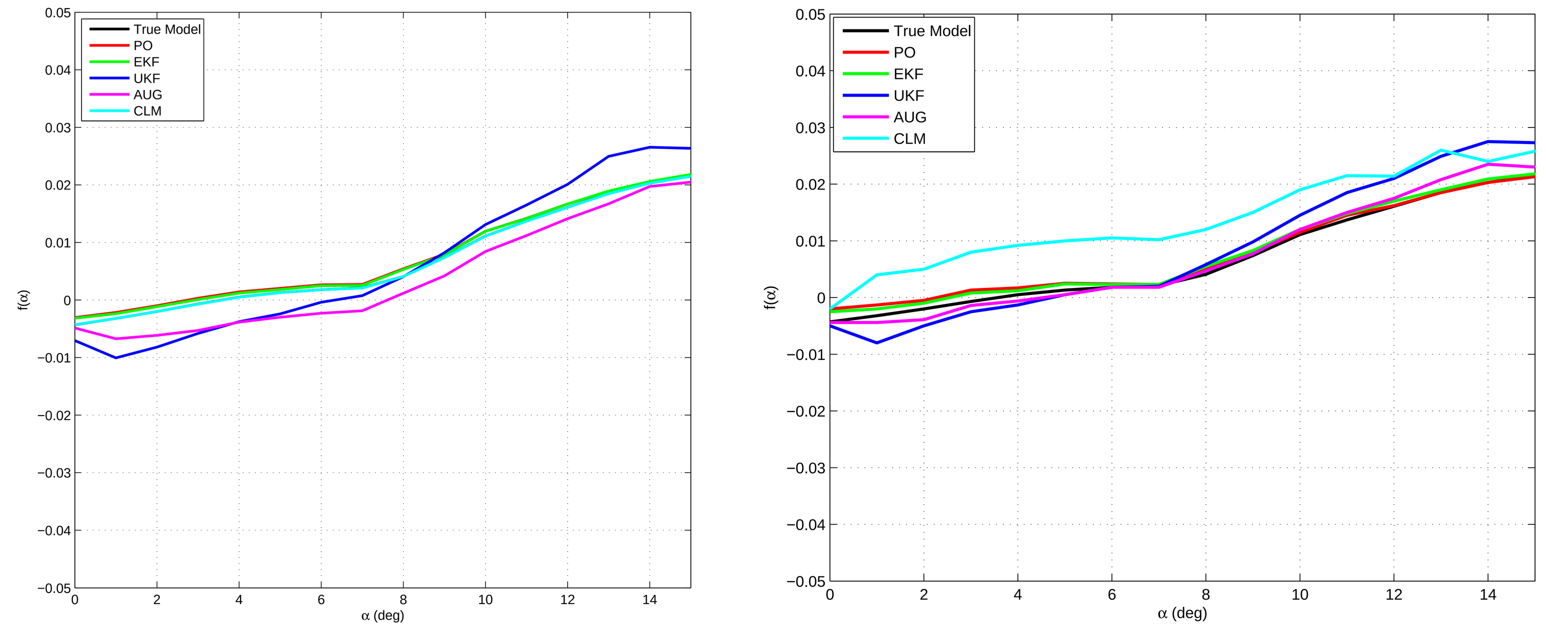
$$\begin{aligned} &\underset{\vartheta}{\text{minimize}}\ V_N(\vartheta, Z^N) \\ &\text{subject to}\ F(\vartheta) = 0. \end{aligned} \tag{8}$$

Results with Simulation Data

Estimation data (left) and validation data (right):

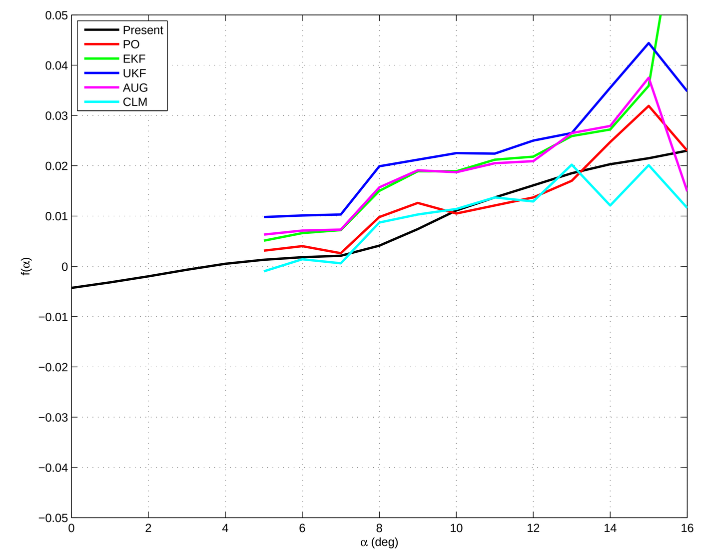


Average nonlinearity estimate for noisy data (left) and for a 10% error in the initial guess (right):



Results with Flight Test Data

Nonlinearity estimate:



- Conclusion: PO and EKF perform best in simulations, PO and CLM perform best on real data.
- Future work: Better handling of process noise, choice of regularization parameter.

Acknowledgments

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