Stability of a piece-wise affine predictor

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Background

The flight characteristics of modern fighter aircraft vary from stable to unstable, from linear to nonlinear, and the flight control system needs to deal with all combinations of these. Also, the process noise characteristics for atmospheric flight is colored, which adds to the system identification complexity. This gives rise to some:

Challenges:

- Nonlinear system
- Closed-loop data
- Partially unknown disturbance characteristics

Engineering constraints:

- Accuracy
- Scalability
- User-independent system identification results

Theory

Flight dynamics can, in general, be described as

$$x_{k+1} = F(x_k, u_k, w_k) \tag{1a}$$

$$y_k = H(x_k, u_k, v_k) \tag{1b}$$

where F describes the nonlinear dynamics of flight and H is the measurement equation. For the aircraft application the measurement is $y_k = x_k + v_k$.

A prediction-error method (PEM) is used for the system identification:

$$\hat{x}_{k+1}(\theta) = F_m(\hat{x}_k(\theta), u_k, \theta) + K_k(\theta)(y_k - \hat{x}_k(\theta)) \tag{2}$$

In this approach the observer gain $K_k(\theta)$ is a parameter to be defined.

$$\theta = \left[\theta_f^T \ \theta_K^T\right]^T \tag{3}$$

Lyapunov stability for discrete-time systems can be summarized as

$$\begin{aligned} V(x^*) &= 0, \quad x^* \text{ is an equilibrium point} \\ V(x) &> 0, \quad \forall x \in \Omega, x \neq x^*, \Omega \subset \Re \\ V(x_{k+1}) - V(x_k) &\leq 0 \\ V(x) &\to \infty \text{ as } \|x\| \to \infty \end{aligned} \tag{4}$$

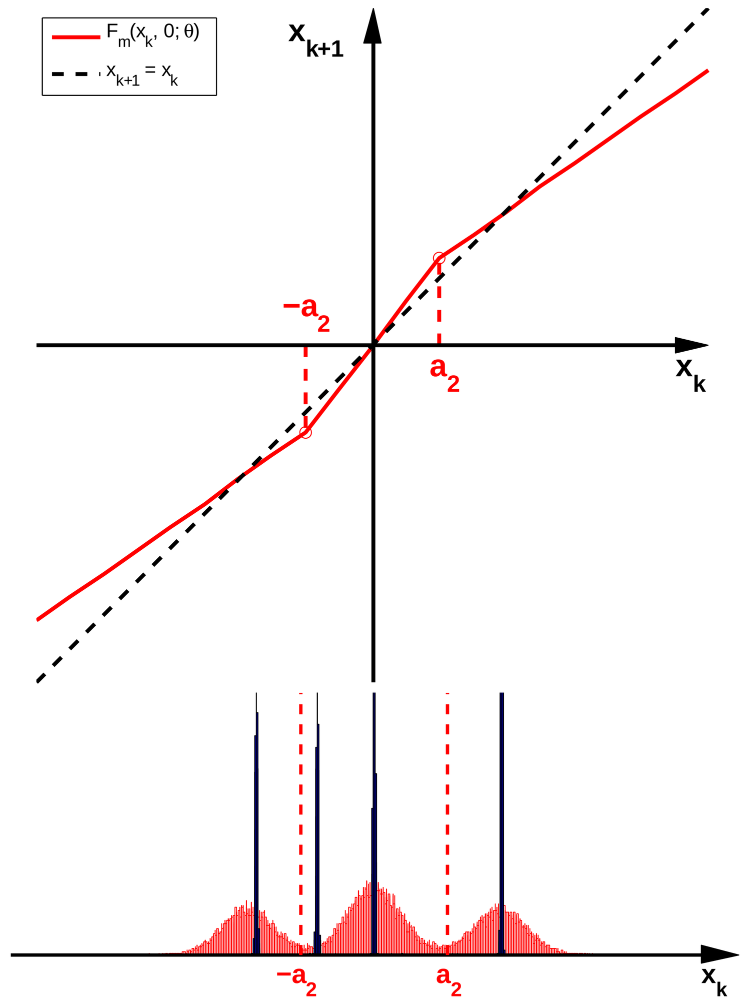
- Can the predictor be globally stable for a constant $K_k(\theta)$?
- Is it possible to get accurate enough estimation results?

Example

For initial stability analyses, a simple scalar example of the predictor in (2) and (3) is used.

$$\begin{aligned} \hat{x}_{k+1}(\theta) &= F_m(\hat{x}_k(\theta), u_k, \theta) + K_k(\theta)(y_k - \hat{x}_k(\theta)) \\ F_m(\hat{x}_k(\theta), u_k, \theta) &= A_1 x_k + A_2(|x_k + a_2| - |x_k - a_2|) + u_k \\ \theta &= \left[A_1 \ K\right]^T \end{aligned} \tag{5}$$

Here, the nonlinearity F_m is a piece-wise affine function with true values $A_1 = 0.7$, $A_2 = 0.3$ and $a_2 = 0.2$. A simple switching feedback has been used to stabilize the system.



Histogram of the x samples with black for measurement noise only and red for measurement + colored process noise.

Results

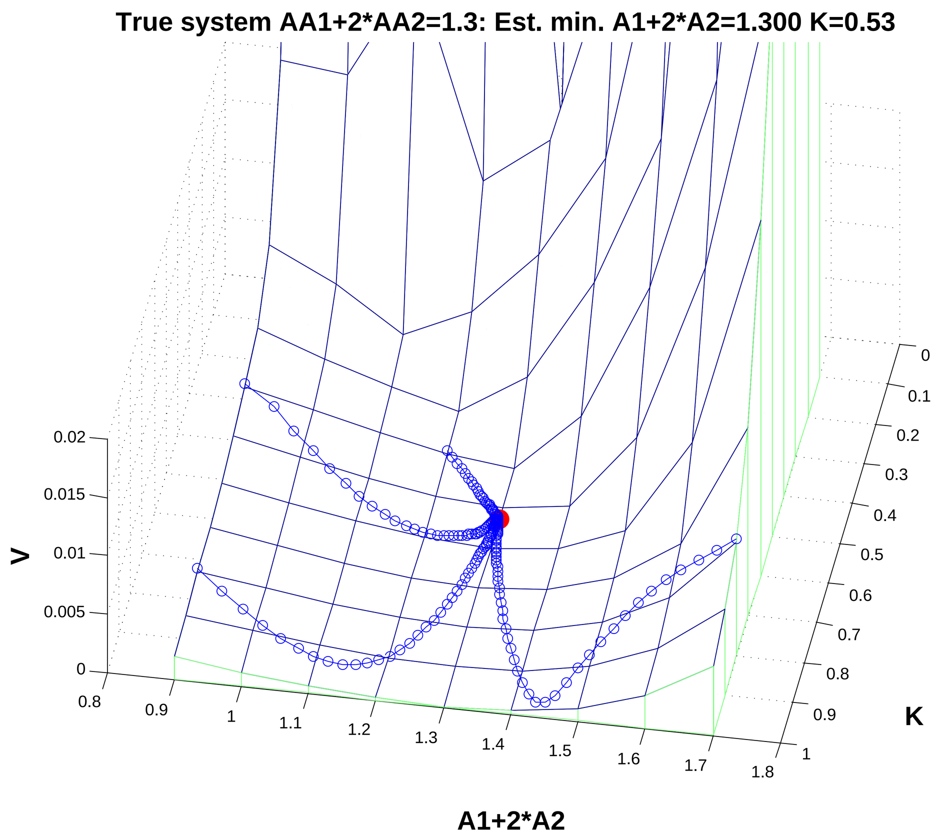
For the example, the following Lyapunov function is used

$$V(\hat{x}_k) = \hat{x}_k^T \hat{x}_k \tag{6}$$

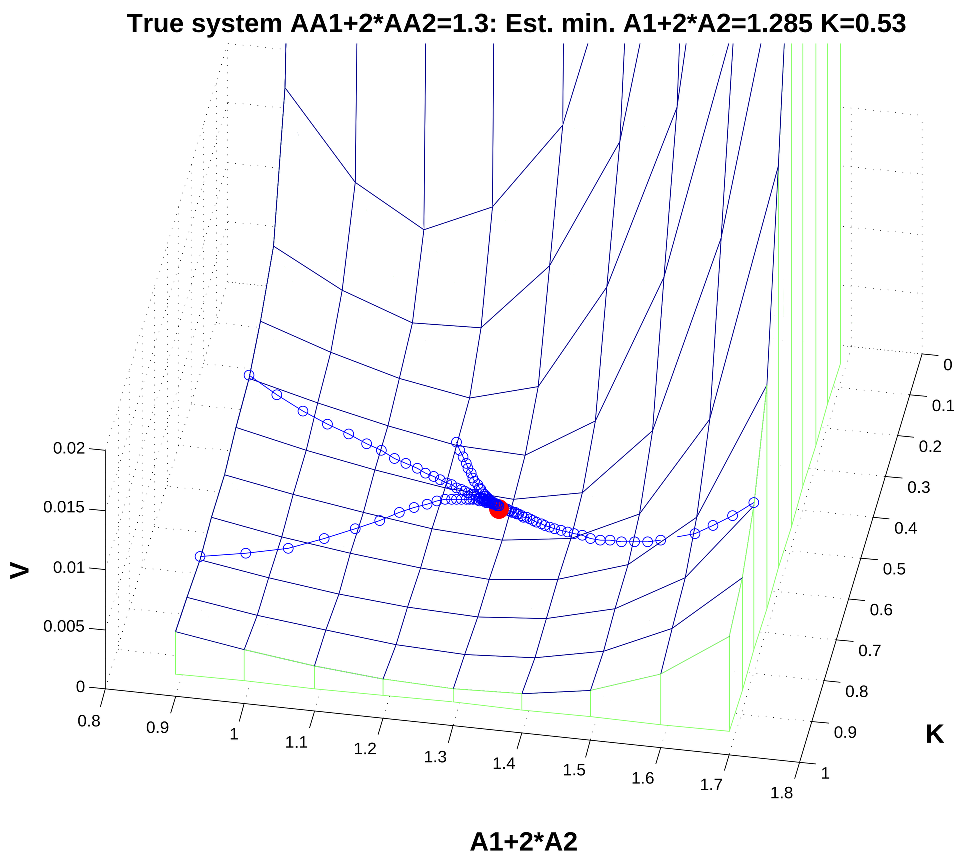
which gives the following two local stability conditions:

$$|(A_1 - K_k(\theta))\hat{x}_k + 2A_2\hat{x}_k| \leq |\hat{x}_k|, \ |\hat{x}_k| \leq a_2 \tag{7a}$$

$$|(A_1 - K_k(\theta))\hat{x}_k + 2A_2 a_2| \leq |\hat{x}_k|, \ |\hat{x}_k| > a_2 \tag{7b}$$



White measurement noise.



White measurement noise + colored process noise.

- There are cases for which global stability of the predictor can be guaranteed using a constant observer gain.
- If the affine parts of the nonlinear function are too different there are cases where the stability cannot be guaranteed.
- The accuracy of the tested example is good enough.
- Evaluations on real data show that the method can be useful also when there are more than one state.

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