Indirect Input Measurements in Dynamic Networks

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Summary

In many cases, it is **intractable or undesirable to measure all variables** in a network and thus, to estimate the complete dynamics. Here, we will show an approach to estimate a part of the network dynamics when **some of the necessary variables are unavailable**. The approach relies on **additional measurements** that are dependent on the unavailable variables and thus **indirectly contain information** about them.

Dynamic Networks

The network consist of L nodes w_k that are affected by external controllable signals r_k and disturbances v_k. Note that v_k or r_k could be zero for all times. These nodes are dynamically dependent on each other and the j^{th} node is described by

$$w_j = \sum_{k \in \mathcal{N}_j} G_{jk}(\mathsf{q}) w_k + r_j + v_j$$

where q is the shift operator and $G_{jk}(\mathsf{q})$ is a transfer function. Here, large calligraphic letters correspond to sets of indices.

- $\mathcal{N} = \{1, \ldots, L\}$: the set of indices of all nodes
- $\mathcal{N}_j$: the set of indices $k \in \mathcal{N} \setminus \{j\}$ where $G_{jk}(\mathsf{q}) \neq 0$
- $\mathcal{S}$: the set of indices of nodes available to the user
- $\mathcal{Z}$: the set of indices of nodes **not** available to the user

The goal is to find an estimate of $G_{ji}(\mathsf{q})$. For brevity, we call r_j and $w_k,\ k \in \mathcal{N}_j$, predictor inputs to w_j and $w_\mathcal{X}$ is the vector containing all $w_k, k \in \mathcal{X}$. This poster concerns a network with a specific structure that using vector notation is described by

$$w_\mathcal{D} = G_{\mathcal{D}\mathcal{D}}(\mathsf{q}) w_\mathcal{D} + G_{\mathcal{D}\mathcal{Z}}(\mathsf{q}) w_\mathcal{Z} + r_\mathcal{D} + v_\mathcal{D} \quad (1)$$
$$w_\mathcal{I} = G_{\mathcal{I}\mathcal{D}}(\mathsf{q}) w_\mathcal{D} + G_{\mathcal{I}\mathcal{I}}(\mathsf{q}) w_\mathcal{I} + G_{\mathcal{I}\mathcal{Z}}(\mathsf{q}) w_\mathcal{Z} + r_\mathcal{I} + v_\mathcal{I} \quad (2)$$
$$w_\mathcal{Z} = G_{\mathcal{Z}\mathcal{D}}(\mathsf{q}) w_\mathcal{D} + G_{\mathcal{Z}\mathcal{Z}}(\mathsf{q}) w_\mathcal{Z} + r_\mathcal{Z} + v_\mathcal{Z} \quad (3)$$

where $G_{\mathcal{D}\mathcal{D}}(\mathsf{q})$, $G_{\mathcal{I}\mathcal{I}}(\mathsf{q})$ and $G_{\mathcal{Z}\mathcal{Z}}(\mathsf{q})$ are zero on the diagonal. Furthermore the set $\mathcal{S}$ has been divided into

- $\mathcal{D} = \mathcal{S} \cap (\mathcal{N}_j \cup \{j\})$: the set of indices of the available predictor inputs to w_j, including the index of w_j
- $\mathcal{I} = \mathcal{S} \setminus \mathcal{D}$: the set of indices of additional variables

Here, it is assumed that $G_{\mathcal{D}\mathcal{D}}$ contains the desired dynamics.

The Immersed Network

The immersed network is the equivalent network, from all external signals r and v to the remaining nodes $w_\mathcal{S}$, when the variables $w_\mathcal{Z}$ not used in the estimation are eliminated. Under certain assumptions, (3) can be solved for $w_\mathcal{Z}$ and inserted into (1) and (2) which after manipulations gives

$$\begin{bmatrix} w_\mathcal{D} \\ w_\mathcal{I} \end{bmatrix} = \begin{bmatrix} \breve{G}_{\mathcal{D}\mathcal{D}}(\mathsf{q}) & 0 \\ \breve{G}_{\mathcal{I}\mathcal{D}}(\mathsf{q}) & \breve{G}_{\mathcal{I}\mathcal{I}}(\mathsf{q}) \end{bmatrix} \begin{bmatrix} w_\mathcal{D} \\ w_\mathcal{I} \end{bmatrix} + \breve{F}(\mathsf{q})(r + v)$$

where $\breve{G}_{\mathcal{D}\mathcal{D}}(\mathsf{q})$ and $\breve{G}_{\mathcal{I}\mathcal{I}}(\mathsf{q})$ are zero on the diagonal. Note that the immersed network typically will depend on the dynamics of the eliminated variables. Potentially, more of the network dynamics has to be modeled compared to when $\mathcal{N}_j \cup \{j\} \in \mathcal{S}$.

Indirect Input Measurements

If $w_\mathcal{I}$ contains information for all missing nodes, an alternative is to solve (2) for $w_\mathcal{Z}$ and inserting it into (1). This gives

$$w_\mathcal{D} = \mathring{G}(\mathsf{q}) w_\mathcal{S} + \mathring{F}(\mathsf{q})(r_\mathcal{S} + v_\mathcal{S})$$

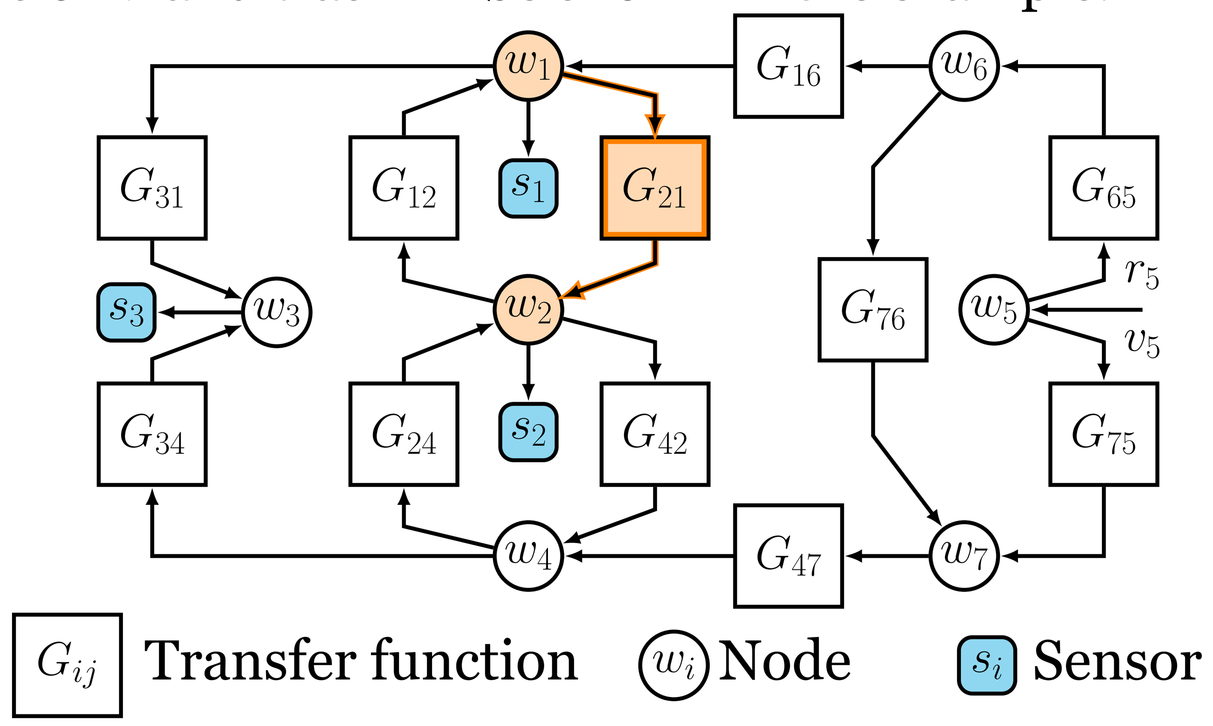
which we call the **indirect model**. It has two benefits:

- It is only dependent on the local variables $w_\mathcal{S}$, $r_\mathcal{S}$ and $v_\mathcal{S}$
- The local variables $w_\mathcal{I}$ contain information about all excitation that enter into the network and have paths to $w_\mathcal{I}$

A partial elimination can be performed even if $w_\mathcal{I}$ does not contain information about all missing nodes and an immersed network can be formed by eliminating the remaining unknown nodes. In this case, the indirect model is

$$w_\mathcal{D} = \tilde{G}(\mathsf{q}) w_\mathcal{S} + \tilde{F}(\mathsf{q})(r + v)$$

Note that this partial elimination can be useful even if it depends on r and v as will be shown in the example.



Illustrative Example

We will illuminate some benefits of the indirect approach by the example shown in the figure and described by

$$\begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \\ w_5 \\ w_6 \\ w_7 \end{bmatrix} = \begin{bmatrix} 0 & G_{12} & 0 & 0 & 0 & G_{16} & 0 \\ G_{21} & 0 & 0 & G_{24} & 0 & 0 & 0 \\ G_{31} & 0 & 0 & G_{34} & 0 & 0 & 0 \\ 0 & G_{42} & 0 & 0 & 0 & 0 & G_{47} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & G_{65} & 0 & 0 \\ 0 & 0 & 0 & 0 & G_{75} & G_{76} & 0 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \\ w_5 \\ w_6 \\ w_7 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ r_5 + v_5 \\ 0 \\ 0 \end{bmatrix}$$

where the goal is to estimate $G_{21}(\mathsf{q})$. The immersed network is

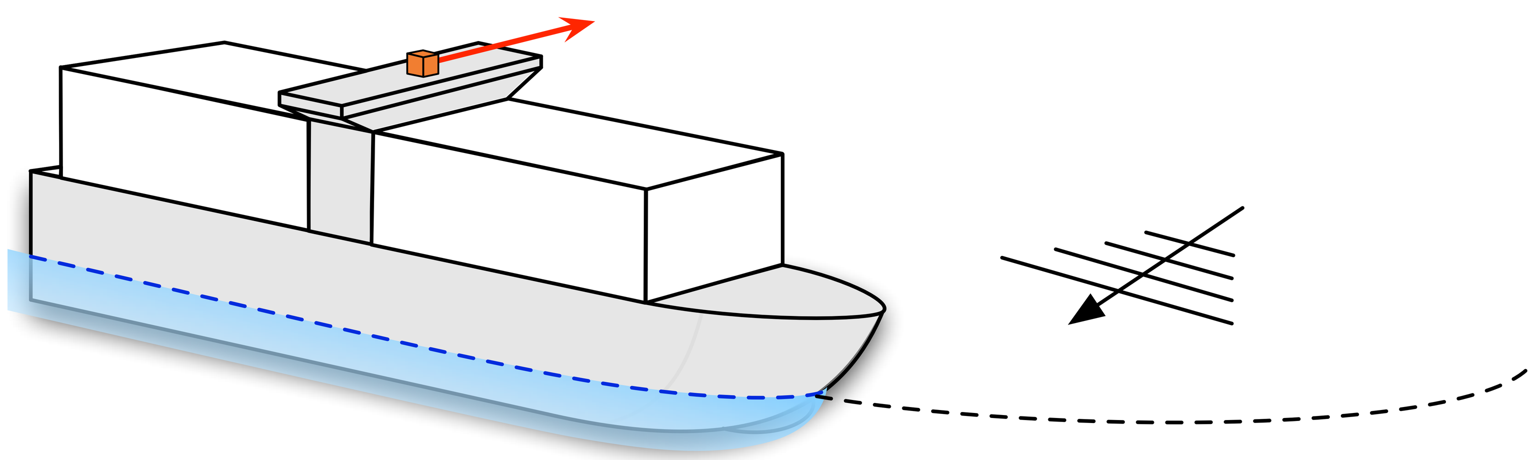
$$\begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \begin{bmatrix} 0 & G_{12} & 0 \\ \frac{G_{21}}{1 - G_{24}G_{42}} & 0 & 0 \\ G_{31} & G_{34}G_{42} & 0 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} + \begin{bmatrix} G_{16}G_{65} \\ \frac{G_{24}G_{47}(G_{75} + G_{76}G_{65})}{1 - G_{24}G_{42}} \\ G_{34}G_{47}(G_{75} + G_{76}G_{65}) \end{bmatrix} (r_5 + v_5)$$

In general, an estimate of $\breve{G}_{21}(\mathsf{q}) \neq G_{21}(\mathsf{q})$. This is due to the interaction between w_2 and w_4. One issue is that v_5 enters directly into all remaining nodes. This could give a bias since v_5 is not available and act as a confounding variable.

The alternative indirect model is given by

$$\begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} 0 & G_{12} & 0 \\ G_{21} - G_{24}G_{34}^{-1}G_{31} & 0 & G_{24}G_{34}^{-1} \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} + \begin{bmatrix} G_{16}G_{65} \\ 0 \end{bmatrix} (r_5 + v_5)$$

Note that w_2 only depends on $w_\mathcal{S}$ and that w_1 and w_3 captures all excitation from both r_5 and more importantly v_5. Furthermore, it is not necessary to estimate a model from r_5 which means that only a local part of the network is estimated.



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