On Indirect Input Measurements

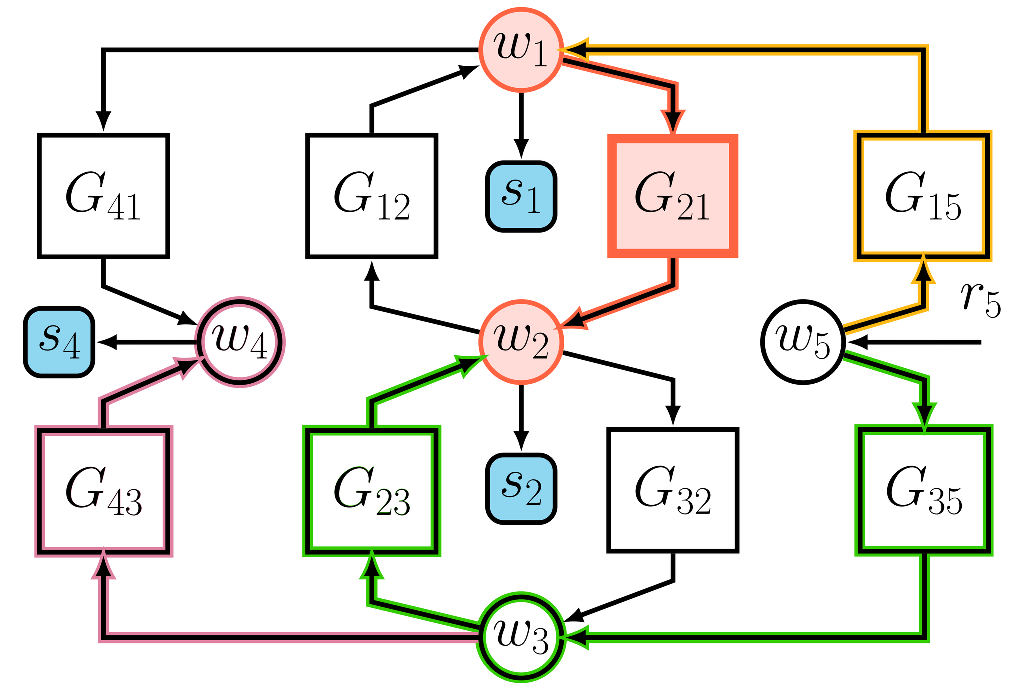
Jonas Linder and Martin Enqvist

Summary

A common issue with many system identification problems is that the **input is unknown**. In this work, a framework is proposed to solve the problem when the input is (partially) unknown and cannot be measured directly. The approach relies on measurements that **indirectly contain information** about the **unknown input**. The resulting indirect model formulation can be used to estimate the desired model of the original system.

Motivational Example

Consider the example of a dynamic network below.



- G_{ij} Dynamic subsystem from the signal w_j to w_i
- w_i The signal w_i = sum of the incoming signals
- s_i The sensor s_i measuring the signal w_i

Note that

- Only some signals are observed
- It is intractable to estimate a complete model
- Instead, interested in G_{21} from w_1 to w_2

Using only the measurements s_1 and s_2 to directly estimate G_{21} from $w_2 = G_{21}w_1 + \tilde{r}$ will lead to a biased estimate since $w_2 = G_{21}w_1 + G_{23}w_3$ is correlated with r_5 both through w_1 (yellow path) and w_3 (green path).

However, there is a measurement of w_4 which is affected by w_3 (purple path) and hence, w_4 **indirectly contains information** about the needed unknown signal w_3. The signal w_4 can then be seen as an **input measurement** to the reformulated model $w_2 = \tilde{G}_{21}w_1 + \tilde{G}_{24}w_4$.

The Indirect Model

Consider

$$y_O = G_O u + H_O \tau = G_{OK}u_K + G_{OI}u_I + G_{OD}u_D + H_O\tau \quad (1)$$

where τ is a disturbance and the input has been divided into

- (exactly) known input u_K
- directly measured input u_D
- indirectly measured input u_I

The input is assumed to be given by

$$u = F_\delta \delta + F_\tau \tau$$

where δ is a known user-controllable signal.

The **direct input measurement** is described by

$$y_D = G_{DD}u_D + H_D\tau \quad (2)$$

where G_{DD} is known and invertible.

Similarly, the **indirect input measurement** is given by

$$y_I = G_{IK}u_K + G_{II}u_I + G_{ID}u_D + H_I\tau \quad (3)$$

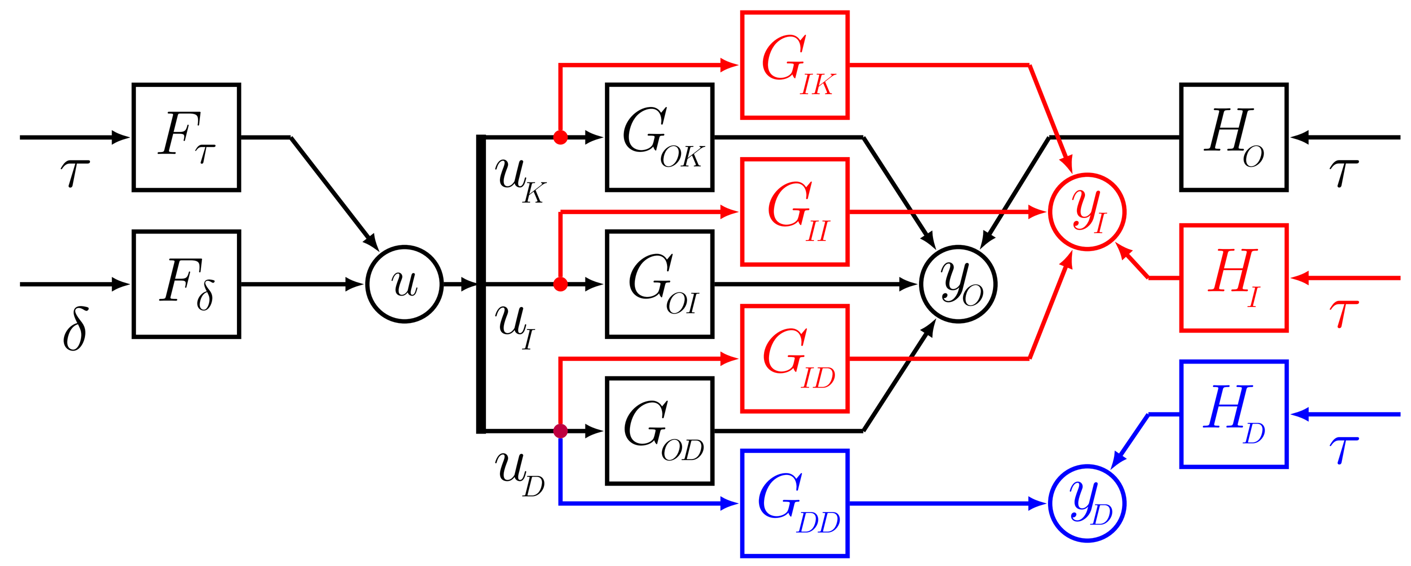
where G_{II} is invertible.

Now, (2) and (3) can be used to eliminate the unknown inputs in (1) which give the **indirect model**

$$y_O = \tilde{G}_{OK}u_K + \tilde{G}_{OI}y_I + \tilde{G}_{OD}y_D + \tilde{\tau} = \tilde{G}_O\tilde{u} + \tilde{\tau} \quad (4)$$

where

$$\tilde{G}_{OI} = G_{OI}G_{II}^{-1},\ \tilde{G}_{OK} = G_{OK} - \tilde{G}_{OI}G_{IK},\ \text{and}\ \tilde{G}_{OD} = \left[G_{OD} - \tilde{G}_{OI}G_{ID}\right]G_{DD}^{-1}.$$



Motivational Example Revisited

For the example, the signals of this framework are given by

$$u_I = w_3,\quad u_D = w_1,\quad y_O = s_2,\quad y_I = s_4,\quad y_D = s_1,$$

which results in the indirect model

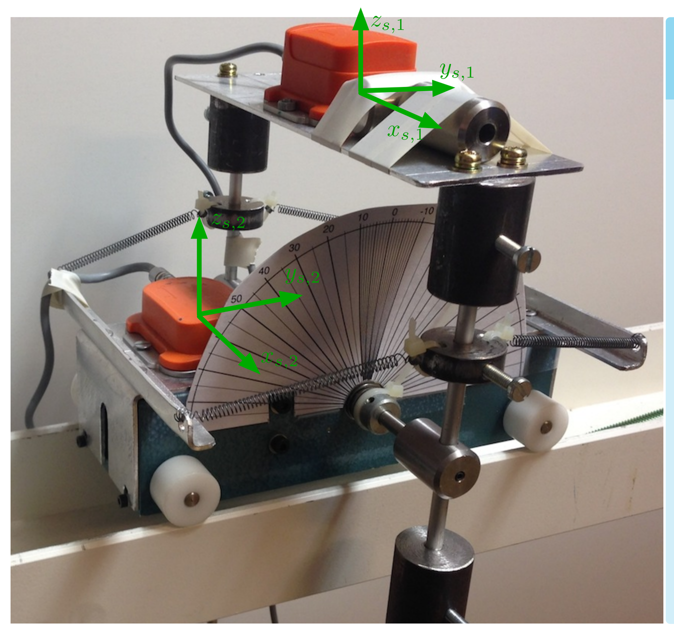
$$s_2 = \left(G_{21} - G_{23}G_{43}^{-1}G_{41}\right)s_1 + G_{23}G_{43}^{-1}s_4$$

Estimation of the Indirect Model (4)

- The input $\tilde{u}$ can be **correlated** with the **disturbance** τ
- The **loop gain** from y_I to y_O might contain a **direct term**

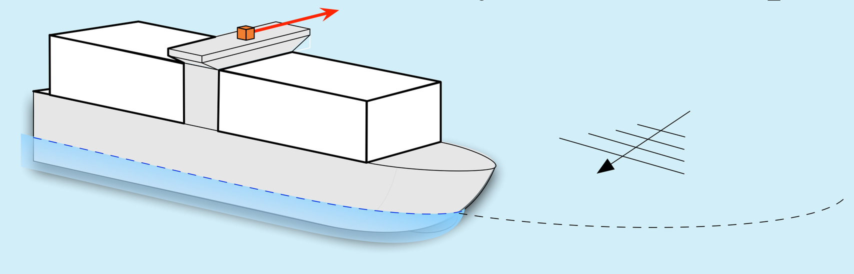
One suitable method is an **iterative instrumental variable** method with instruments simulated from δ.

Experimental Verification



The Modified Inverted Pendulum

The modified inverted pendulum might at a first glance seem trivial. However, it is a good approximation of many systems, such as the roll dynamics of ships.



For the pendulum above, the goal is to estimate the change in **mass** m and change in **center of mass** z_m using measurements of the pendulum's motion. A model from the cart acceleration to the angle of the pendulum is given by

$$\phi = \frac{b_0(m,z_m)}{\mathsf{p}^2 + a_1(m,z_m)\mathsf{p} + a_2(m,z_m)}a_y = G(\mathsf{p})a_y$$

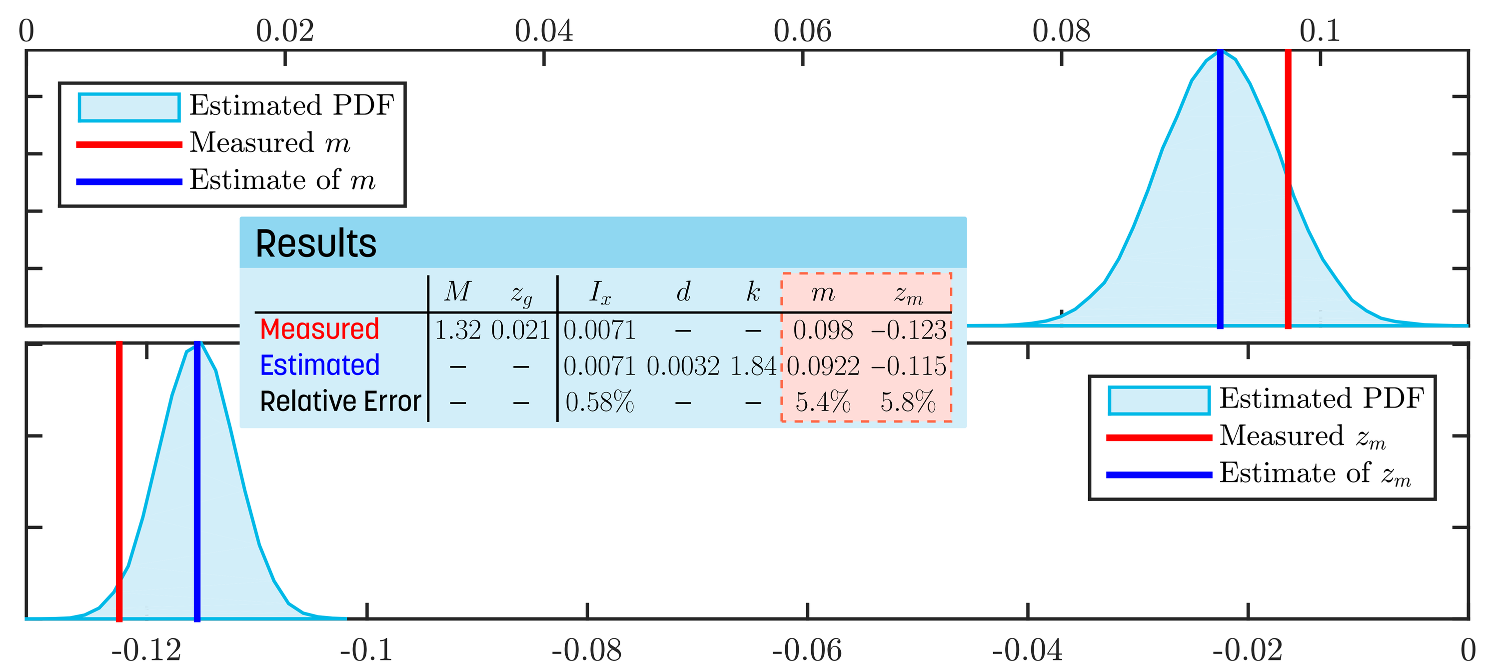
The input a_y is unknown but indirectly measured

$$y_I = z_s\ddot{\phi} + \phi g - a_y = G(\mathsf{p})\left[(z_s\mathsf{p}^2 + g) - 1\right]a_y = G_{II}a_y$$

which combined with $y_O = \dot{\phi}$ give the indirect model

$$y_O = G(\mathsf{p})\mathsf{p}\,G_{II}^{-1}y_I = \tilde{G}_{OI}y_I = \frac{\beta_0(m,z_m)\mathsf{p}}{\mathsf{p}^2 + \alpha_1(m,z_m)\mathsf{p} + \alpha_2(m,z_m)}y_I$$

Estimates of m and z_m using the indirect modeling approach and data from the pendulum can be seen below.



Results

	M	z_g	I_x	d	k	m	z_m
Measured	1.32	0.021	0.0071	–	–	0.098	−0.123
Estimated	–	–	0.0071	0.0032	1.84	0.0922	−0.115
Relative Error	–	–	0.58%	–	–	5.4%	5.8%

LINKÖPING UNIVERSITY
LINK-SIC

LINKÖPING UNIVERSITY
Division of Automatic Control