Tool Position Estimation for a Flexible Manipulator

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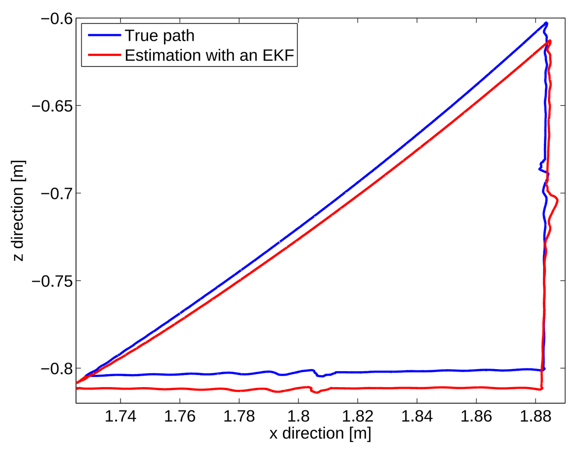
Background

The **problem** is to estimate the tool position for a flexible manipulator. The manipulator is a *resonant system* with *uncertainties* in the model parameters. There are also *high demands* on the accuracy of the estimation. Earlier work, see [1], has shown that the estimation is good for frequencies from 3 to 30 Hz but not so good for lower frequencies. The **aim** of this work is therefore to improve the estimation in the low frequency range.



Models

A nonlinear two degrees of freedom **robot model** is used:

$$\dot{x} = f(x,u) = \begin{pmatrix} x_3 \\ x_4 \\ M^{-1}(x_1)(u - C(x) - G(x_1) - D(x) - \tau_s(x) - \kappa(x_4)) \end{pmatrix}$$

where $x = \begin{pmatrix} q_a & q_m & \dot{q}_a & \dot{q}_m \end{pmatrix}^T$. The measured acceleration in frame $\{s\}$ fixed to the sensor gives an **acceleration model**:

$$\ddot{\rho}_s^M = \ddot{\rho}_s + R_s^w(q_a)G_w + \delta_s + e_s.$$

$\ddot{\rho}_s$ is calculated as $R_s^w(q_a)\ddot{\rho}_w$, where $\ddot{\rho}_w$ is the second derivative of the vector ρ_w with respect to time. ρ_w is a vector from the origin of frame $\{w\}$ to the origin of frame $\{s\}$ expressed in frame $\{w\}$.

Notation

$M(q)$	Inertia matrix
$C(q,\dot{q})$	Coriolis- and centrifugal terms
$G(q)$	Gravitaion torque
$\tau_s(q)$	Nonlinear stiffness torque
$D(\dot{q})$	Damping torque
$\kappa(\dot{q})$	Nonlinear friction torque
$\ddot{\rho}_s$	Acceleration from the motion
$R_s^w(q_a)$	Rotation matrix from $\{w\}$ to $\{s\}$
G_w	Gravitation in $\{w\}$
δ_s	Drift
e_s	Measurement noise

Observer

An Extended Kalman Filter, *EKF*, is used to estimate the position of the robot. Euler forward is used to discretize the state space model according to

$$x_{k+1} = F(x_k, u_k) + v_k, \quad F(x_k,u_k) = x_k + T_s f(x_k,u_k)$$

The measurements are motor angles and sensor acceleration and are expressed as

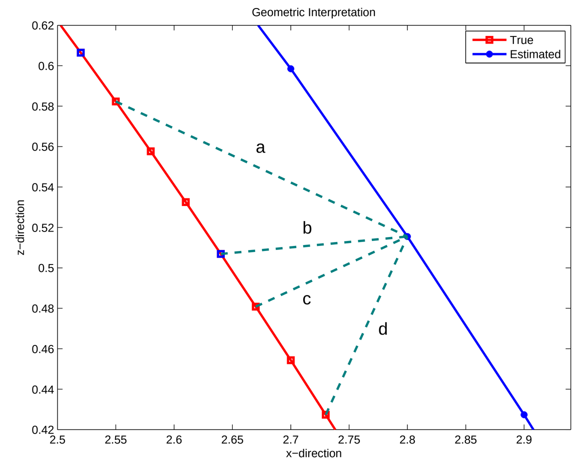
$$z_k = h(x_k,u_k) + w_k = \begin{pmatrix} x_{2k} \\ R_s^w(x_{1k})(\ddot{\rho}_w(x_k) + G_w) \end{pmatrix} + w_k.$$

Covariance Optimization

The problem is to choose the covariance matrices for the observer such that the path error is minimized. The path error is defined as

$$e_k = \min_i \sqrt{|p_{x,i} - \hat{p}_{x,k}|^2 + |p_{z,i} - \hat{p}_{z,k}|^2},$$

where $p_{x,i}$, $\hat{p}_{x,k}$, $p_{z,i}$ and $\hat{p}_{z,k}$ are the true and estimated position for the tool in the x- and z-direction at time k and time i, respectively. A qubic spline interpolation to get data points between the estimated points is required. The optimization problem can now be summarized as



$$\text{Minimize } f_{obj}(\hat{p}_x,\hat{p}_z) = \sqrt{\sum_{k=1}^N |e_k|^2}$$

$$\text{subject to } \lambda_j > 0 \quad j = 1,\dots,5$$

$$\widetilde{Q}_\lambda = \begin{pmatrix} \lambda_1 I_{2\times2} & 0 & 0 & 0 \\ 0 & \lambda_2 I_{2\times2} & 0 & 0 \\ 0 & 0 & \lambda_3 I_{2\times2} & 0 \\ 0 & 0 & 0 & \lambda_4 I_{2\times2} \end{pmatrix} \widetilde{Q}$$

$$\widetilde{R}_\lambda = \begin{pmatrix} \lambda_5 I_{2\times2} & 0 \\ 0 & I_{2\times2} \end{pmatrix} \widetilde{R}$$

$$(\hat{p}_x,\hat{p}_z) = \text{EKF}(\widetilde{Q}_\lambda, \widetilde{R}_\lambda)$$

where λ_j are the optimization parameters. $\widetilde{Q}$ and $\widetilde{R}$ are diagonal matrices with the elements taken from the covariances for the process noise v and the measurement noise w.

Simulation Setup

Three types of simulations are executed on 4 different paths. A set of covariance matrices are then optimized for each simulation.

Sim1: Without errors

Sim2: With calibration errors, drift and model errors

Sim3: With calibration errors, drift and without model errors

Cov1: Optimized for Sim1 on Path A (Red)

Cov2: Optimized for Sim2 on Path A (Green)

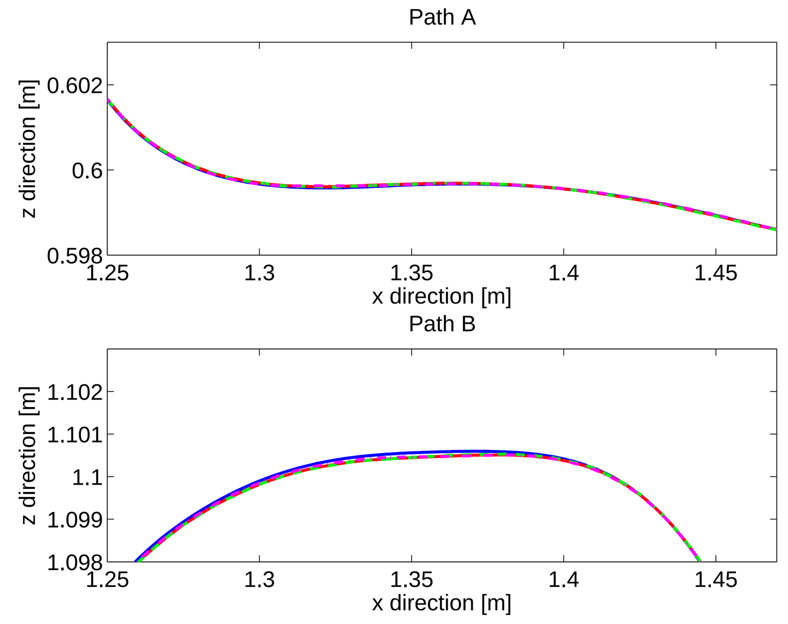
Cov3: Optimized for Sim3 on Path A (Magenta)

Result

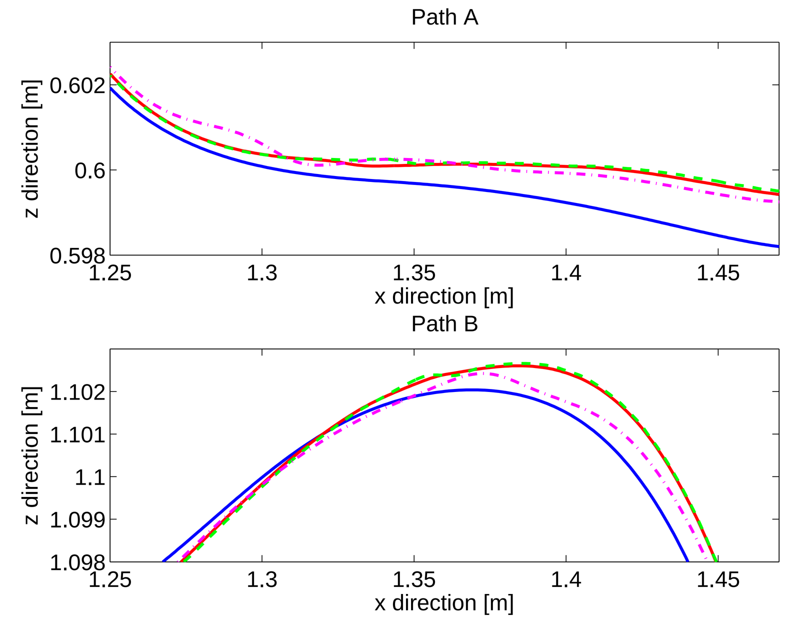
All 9 combinations of the simulations and the covariance matrices are used to evaluate the performance of the observer.

- Small estimation errors for all 3 set of covariance matrices in Sim1.
- Difficult to get good estimations when model errors are present.
- Calibration errors and drift do not affect very much.
- The estimation is robust for these 4 paths.

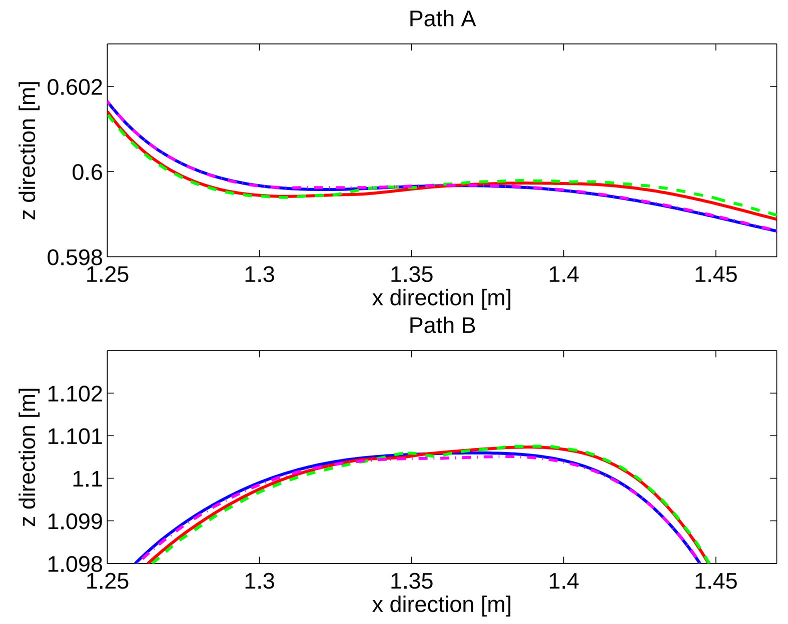
Estimation on Sim1 for three different covariance matrices



Estimation on Sim2 for three different covariance matrices



Estimation on Sim3 for three different covariance matrices



Max and mean error in mm for the EKF on path A

Path A	COV1		COV2		COV3	
	MAX	MEAN	MAX	MEAN	MAX	MEAN
SIM1	0.078	0.025	0.080	0.025	0.080	0.026
SIM2	1.681	0.550	1.577	0.543	1.910	0.661
SIM3	0.400	0.113	0.903	0.172	0.079	0.027

Max and mean error in mm for the EKF on path B

Path B	COV1		COV2		COV3	
	MAX	MEAN	MAX	MEAN	MAX	MEAN
SIM1	0.124	0.035	0.126	0.035	0.112	0.035
SIM2	1.908	0.654	1.966	0.657	2.137	0.687
SIM3	0.419	0.082	0.842	0.120	0.111	0.035

Conclusions

- The offset in the estimation in [1] is not present in simulations.
- The covariance optimization gives different minimum due to different initial values, which affect the estimation.
- The optimization of the covariance matrices is a difficult task.
- The estimation is robust for the paths in the simulation. Next step involves to include paths that better cover the complete robot workspace.

[1] R. Henriksson, M. Norrlöf, S. Moberg, E. Wernholt and T. Schön, *Experimental Comparison of Observers for Tool Position Estimation of Industrial Robots*, 2009, to appear in the 48th IEEE Conference on Decision and Control.