

Summary

- The *flexible joint model* (only gearbox elasticity) is shown to be insufficient for accurate modeling of modern robot manipulators.
- The *extended flexible joint model* (additional non-actuated joints to model elasticity of links and bearings) describes the motor and the tool movements with good accuracy.
- Unknown model parameters are estimated using a *frequency domain gray-box identification method*.
- A method for improving the parameter estimate by compensating input and output dynamics and nonlinearities is also suggested and evaluated.
- Similar model parameters are obtained when using two different output variables, the *motor position* and the *tool acceleration*.
- Tool acceleration measurements seem to reduce the uncertainty in some cases and can probably increase the identifiability of some parameters.
- The extended flexible joint model gives a much harder control problem, where the inverse dynamics problem involves solving a high-index DAE, but might in the future improve robot performance.

Robot Manipulator Model

The *extended flexible joint* model consists of a serial kinematic chain of rigid bodies, connected by multidimensional spring-damper pairs. Each spring-damper pair represents a connection that can either be *actuated* (modeling gearbox elasticity) or *non-actuated* (modeling other elasticities). With Υ_a actuated joints and Υ_{na} non-actuated joints, the system has $\Upsilon = \Upsilon_a + \Upsilon_{na}$ joints and $2\Upsilon_a + \Upsilon_{na}$ degrees-of-freedom (DOF). The model equations are

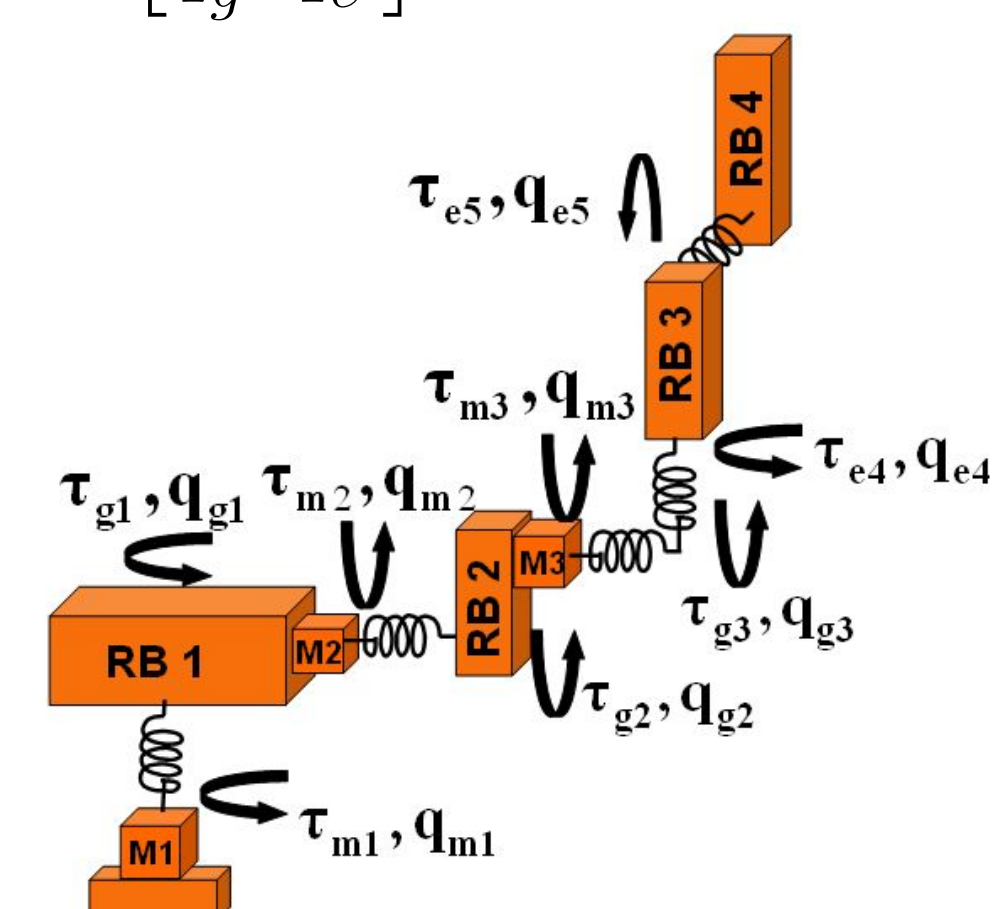
$$M(q_a)\ddot{q}_a + c(q_a, \dot{q}_a) + g(q_a) = \begin{bmatrix} \tau_g \\ \tau_e \end{bmatrix},$$

$$K_g(rq_m - q_g) + D_g(r\dot{q}_m - \dot{q}_g) = \tau_g,$$

$$-K_e q_e - D_e \dot{q}_e = \tau_e,$$

$$M_m \ddot{q}_m + f(\dot{q}_m) = \tau_m - r\tau_g,$$

with $q_a = [q_g^T \ q_e^T]^T$. The *flexible joint model* is a special case if $\Upsilon_{na} = 0$.



An extended flexible joint model with 8 DOF.

Notation:

$q_g \in \mathbb{R}^{\Upsilon_a}$	Actuated joint angular position
$q_e \in \mathbb{R}^{\Upsilon_{na}}$	Non-actuated joint angular position
$q_m \in \mathbb{R}^{\Upsilon_a}$	Motor angular position
$M_m \in \mathbb{R}^{\Upsilon_a \times \Upsilon_a}$	Motor inertia matrix
$M(q_a) \in \mathbb{R}^{\Upsilon \times \Upsilon}$	Inertia matrix for the joints
$c(q_a, \dot{q}_a) \in \mathbb{R}^{\Upsilon}$	Coriolis and centrifugal torques
$g(q_a) \in \mathbb{R}^{\Upsilon}$	Gravity torque
$f(\dot{q}_m) \in \mathbb{R}^{\Upsilon_a}$	Nonlinear friction
r	Gear ratio
$\tau_m \in \mathbb{R}^{\Upsilon_a}$	Actuator torque
$\tau_g \in \mathbb{R}^{\Upsilon_a}$	Actuated joint torque
$\tau_e \in \mathbb{R}^{\Upsilon_{na}}$	Non-actuated joint torque
K_g, K_e, D_g, D_e	Stiffness- and damping matrices

Parameter Estimation

- The number of non-actuated joints and rigid bodies are assumed to be determined by a bottom-up approach (start with a flexible joint model and add non-actuated joints).
- The rigid body parameters for the links are assumed to be known from a CAD model and properly verified on a real manipulator.
- *Estimation problem:* Identify the elasticity and friction parameters.

Identification procedure:

A frequency domain procedure (see [1] for details) based on the *estimation of intermediate local models*:

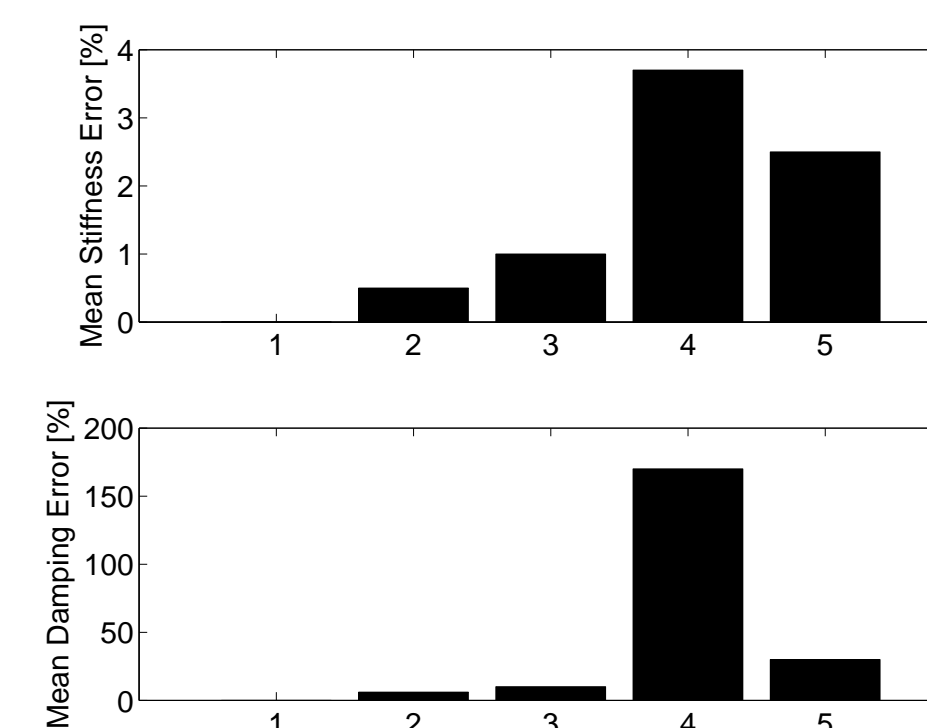
1. Estimate the nonparametric frequency response function (FRF) in a number of operating points.
2. Linearize the nonlinear gray-box model in the same operating points, resulting in parametric FRFs.
3. Obtain the optimal parameters by minimizing the discrepancy between the nonparametric FRFs and the parametric FRFs.

Results

Attainable Accuracy

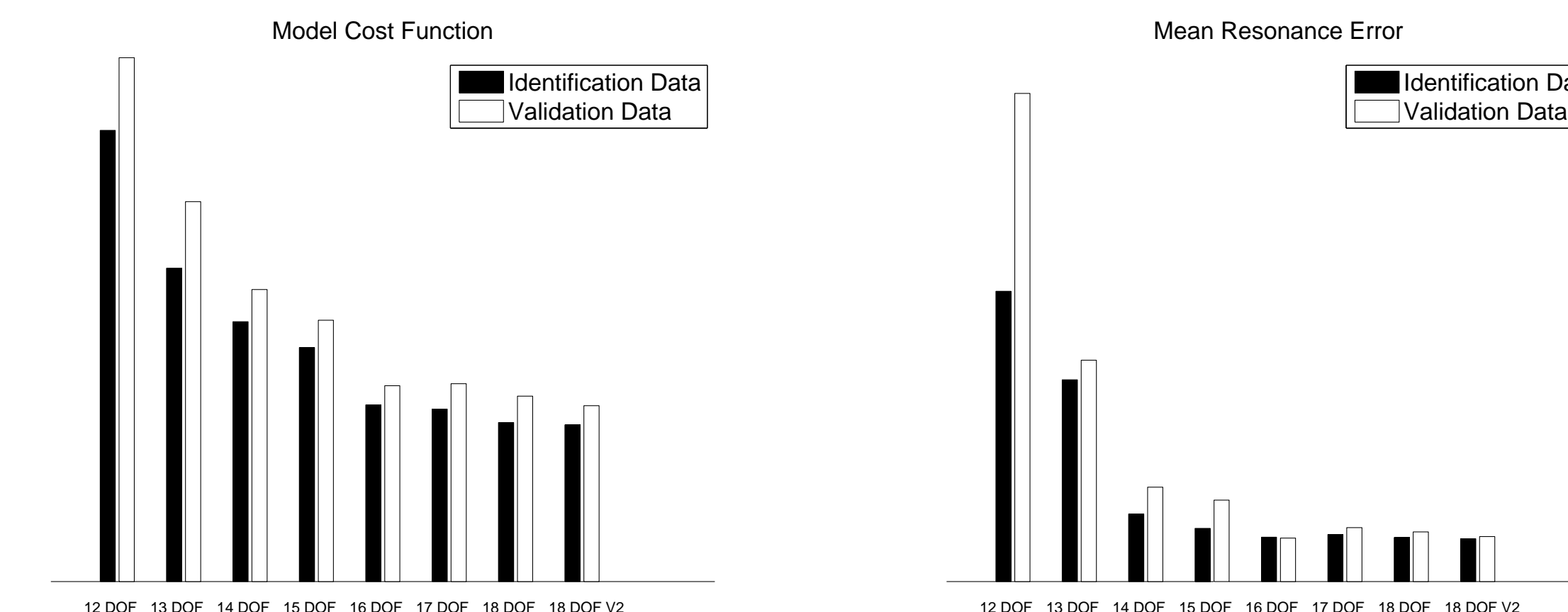
To study the attainable accuracy, a realistic simulation model is used with $\Upsilon_a = 6$, $\Upsilon_{na} = 5$. FRFs are estimated from motor torque τ_m to motor position q_m for 5 different operating points and the estimated spring-damper pairs are studied for the five cases:

1. Linear Model
2. Linear Model + Disturbances
3. Nonlinear model + Disturbances
4. Nonlinear model + Disturbances + Friction
5. Nonlinear model + Disturbances + Friction + "Nonlinear input compensation"

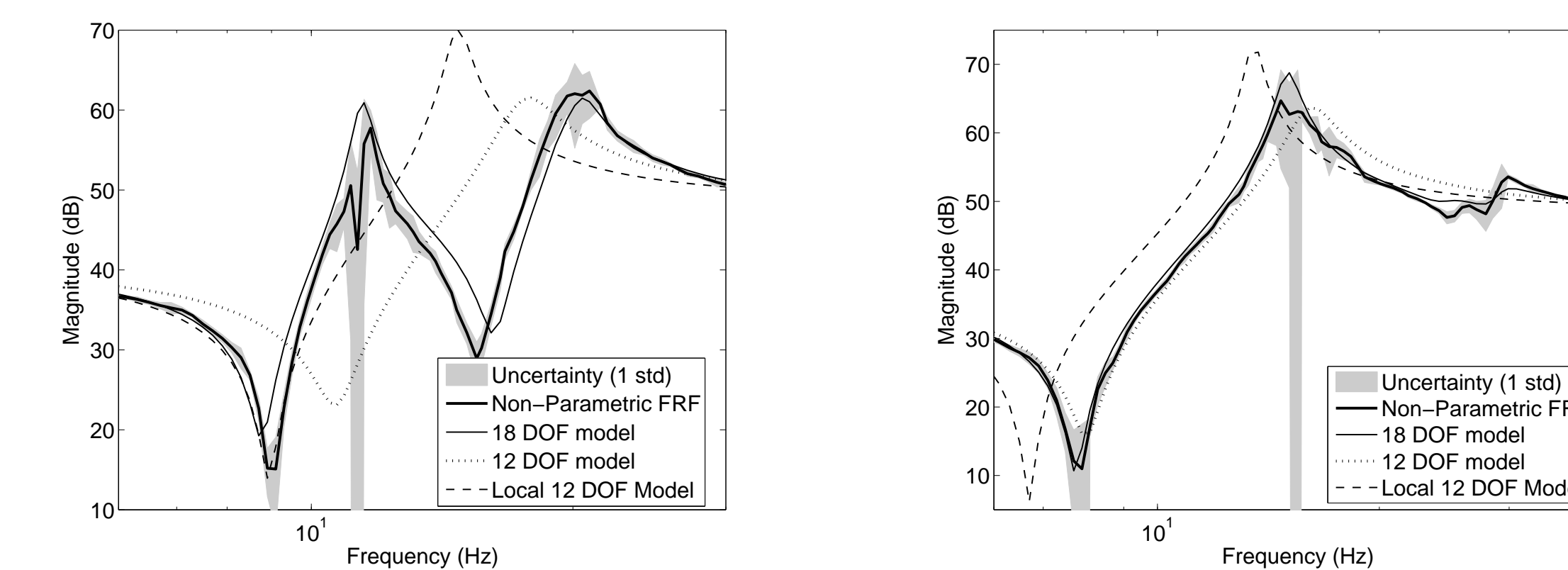


Model Structure Selection and Validation

- Experiments from a modern six-axes medium-sized ABB robot.
- Parameters are *estimated* for different model complexities using FRF estimates in 7 different robot configurations, and *validated* in these as well as in 7 completely different robot configurations.
- Model accuracy is evaluated by comparing the *cost functions* and the *mean frequency error in the dominating pole-zero pair*.



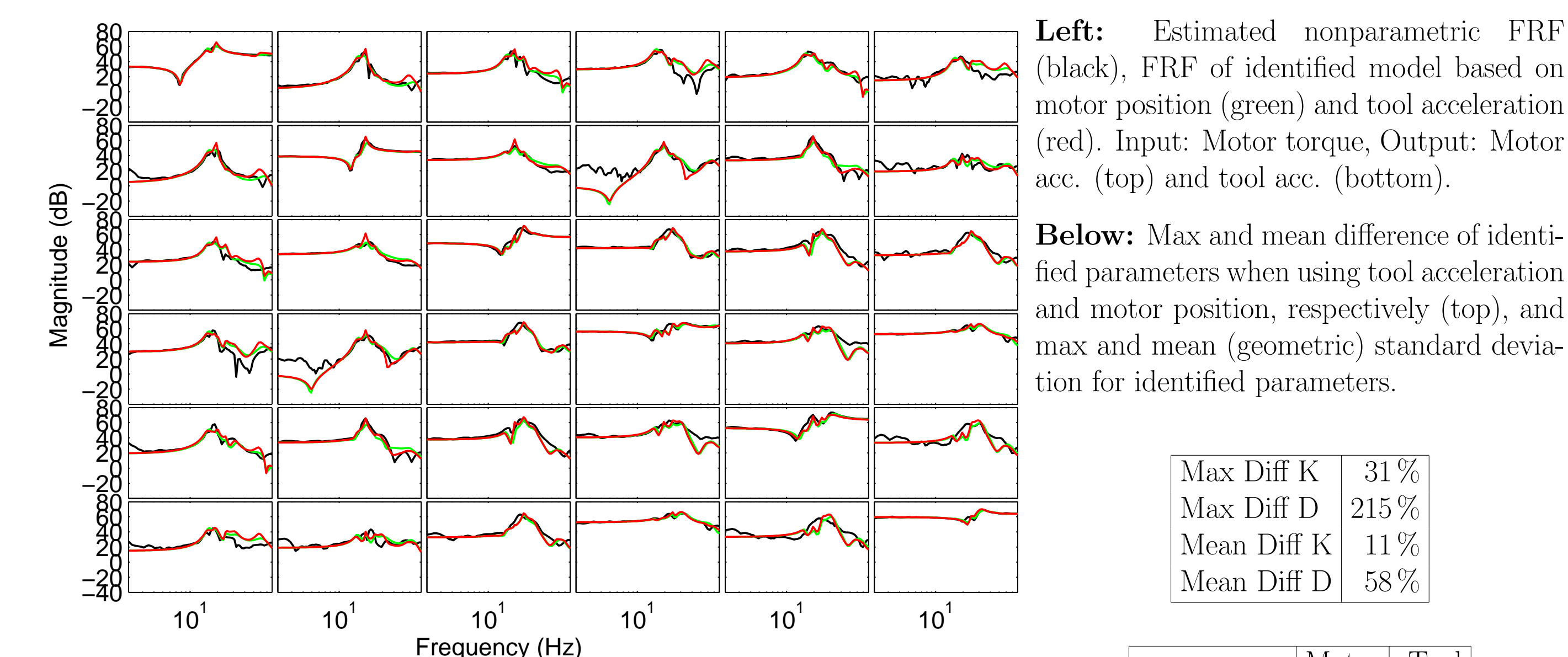
The 12 DOF model (flexible joint model) cannot describe the manipulator. If the 12 DOF model is optimized only for one configuration and for low frequencies (Local 12 DOF Model), the first zero can be modeled, but not the first pole. If this locally optimized model is evaluated in another configuration the zero is not properly modeled.



One FRF element in configuration 1 (left) and 2 (right).

Identification Using Accelerometers

- Tool acceleration measured using a three-axes accelerometer.
- Estimate FRFs to motor position and tool acceleration.
- Motivation: Verify if model describes tool movement and investigate if additional sensors should be used.



Left: Estimated nonparametric FRF (black), FRF of identified model based on motor position (green) and tool acceleration (red). Input: Motor torque, Output: Motor acc. (top) and tool acc. (bottom).

Below: Max and mean difference of identified parameters when using tool acceleration and motor position, respectively (top), and max and mean (geometric) standard deviation for identified parameters.

Max Diff K	31%
Max Diff D	215%
Mean Diff K	11%
Mean Diff D	58%

	Motor	Tool
Max Std K	2.1%	1.6%
Max Std D	19%	33%
Mean Std K	1.6%	1.0%
Mean Std D	8.5%	6.4%

[1] Wernholt, E. and Moberg, S. Nonlinear gray-box identification using local models applied to industrial robots. *Automatica*. Provisionally accepted for publication.